Refurbishing authorization strategy in the secondary market for electrical and electronic products

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ABSTRACT

In the electrical and electronic products secondary market, third party resellers (TPR) engaging in product refurbishing are growing rapidly, which provides direct competition to the original equipment manufacturers' (OEMs) new product sales. Irrespective of prices, consumers have shown a higher preference for refurbished products from the OEMs and OEM-authorized TPRs than non-authorized TPRs. As a result, some OEMs choose to provide refurbishing authorization to TPRs who are required to meet specific standards and pay an authorization fee. Besides the strategy of refurbishing authorization, an OEM could choose strategies such as participating in direct refurbishing work or not at all. The main goal of this paper is to examine under what conditions the refurbishing authorization strategy is optimal for the OEM. The essential tradeoff is whether the indirect benefit from authorizing a TPR, which includes an enlarging market share, can outweigh the direct cannibalization effect of authorized refurbished products on new products sales. To that end, we develop Bertrand competition models between an OEM and a TPR using several strategies. Results show that when the consumers’ preference for refurbished products relative to new products is not large enough, the OEM should choose the authorization strategy. The OEM and authorized TPR achieve a win-win outcome under some conditions. We also find that a higher authorization fee is not always a better option for the OEM.

1. Introduction

With the updating and upgrading of electrical and electronic products (hereafter referred to as e-products), the secondary market has been growing steadily. The secondary market constitutes a complex ecosystem of resellers and consumers who are actively engaging in the sale and purchase of refurbished products. Refurbishing is one form of remanufacturing (Abbey et al., 2015a). Modules disassembled from returned products are cleaned and replaced or restored, and then reassembled into refurbished products (Lund, 1984). According to eBay’s explanations, a refurbished product has been “professionally restored to working order, but may or may not be in the original packaging”. Refurbished products mainly come from secondhand goods, demo units, and those with opened boxes, shipping or exterior damage, or production defects (Swamy, 2014). These products, now restored to as-new condition, go into the secondary market for sale (Guide and Wassenhove, 2001).

In recent years, consumers have increasingly purchased refurbished products in the e-products secondary market. According to a survey implemented in 2014 by Liquidity Services which is a global solutions provider in the reverse supply chain, 94% of 1407 respondents had purchased a refurbished product in the past three years and 81% would recommend buying refurbished products to others (Businesswire, 2014). Price advantage, growing environmental-friendly awareness of consumers, and complete functional performance are the several reasons why refurbished products are welcomed (Nidhi, 2015). The vast market demand has attracted many third-party resellers (TPRs) to embark upon the business of refurbishing (Ferguson and Toktay, 2006). Generally, markets for new and refurbished products will overlap, resulting in competition between these products (Debo et al., 2006; Ferrer and Swaminathan, 2006). Research shows that refurbished products from TPRs in the secondary market will likely cannibalize OEMs’ new products sales to some degree and exert competitive pressure on OEMs (Agrawal et al., 2015; Atasu et al., 2008b; Guide and Wassenhove, 2009).

For most e-products, the existence of an active secondary market is not controlled by manufacturers, so OEMs may choose to cooperate and work with TPRs (Xiong et al., 2016). Refurbishing authorization is one
form of cooperation and has been adopted by some OEMs. OEMs provide refurbishing authorization to TPRs and equip their refurbished products with manufacturer-authorized signs. For example, there are "manufacturer or manufacturer-approved refurbished" products and regular "seller refurbished" products on sale on eBay (Ebay, 2017). OEMs, such as HP and Lenovo, outsource refurbishing and reselling to manufacturer-authorized TPRs. This is the quote from the Sustainability: Reuse and Recycling at HP website: “We work with closely vetted reuse and recycling vendors to ensure environmentally responsible and high value recovery options.” (Hewlett-Packard, 2017). Since 1987, HP has recovered 1,683,000 tons of computer hardware (for reuse and recycling) and HP supplies (for recycling). Lenovo also has an audit program to inspect recycling partners to ensure conformance with its specific policies and standards (Lenovo, 2017).

Besides authorizing a TPR to carry out refurbishing work, which is considered as strategy 1, the OEM has two more strategies. Strategy 2 is the OEM engaging in refurbishing itself, and strategy 3 is the OEM not involved in refurbishing work at all (Souza, 2013). For example, Dell and Apple mainly carry out their own OEMs' new work and sell refurbished products on their official websites (Apple, 2017; Dell, 2017). We examine the conditions under which the refurbishing authorization strategy is optimal for OEMs, compared with strategy 2 and 3.

Consumers' preference for non-authorized refurbished products and authorized refurbished products will affect the market share of different products, which further determines whether refurbishing authorization is a win-win strategy for an OEM and a TPR. Consumers who consider purchasing second-hand products are quite sensitive to the quality of products (Wang and Hazen, 2016) and warranty service (Zhu et al., 2016). Irrespective of prices, consumers usually show a higher preference for the manufacturers' or manufacturer-authorized refurbished products than non-authorized ones. This is because manufacturers' or manufacturer-authorized TPRs have to refurbish according to best practices, thus product quality and service can be guaranteed to some extent. On eBay, authorized refurbished products “have been inspected, cleaned, and repaired to meet manufacturer specifications and is in excellent condition” (Ebay, 2017). Therefore, consumers are willing to pay a higher price for the authorized refurbished products due to brand credibility and warranty service. The paper assumes that consumers' preference for manufacturer or manufacturer-authorized refurbished products is 1, and for non-authorized channel is β/β < 1. This parameter setting has been adopted by Ferrer and Swaminathan (2006).

In the refurbishing authorization strategy, the essential tradeoff for an OEM is whether the benefit from authorizing TPRs, which includes enjoying brand advantage, enlarging market share, and increased profitability from price discrimination, can outweigh the cannibalization effect of authorized refurbished products on new products sales. When an OEM authorizes a TPR, the TPR's refurbished products will have a higher consumer preference. As an investment return, an authorization fee is paid to the OEM by the authorized TPR (Oraiopoulos et al., 2012; Zou et al., 2016), which fully exploits the OEM's brand advantage. Moreover, selling refurbished products is similar to adding low-level products, so an OEM could expand its market share and benefit from price discrimination. However, there is competition between new and refurbished products, which could lead to sales cannibalization.

Previous researches have mainly focused on whether and how OEMs can discourage third-party entrants because of the cannibalization effect. Oraiopoulos et al. (2012) point out that the resale effect that secondary market brings will reduce the sales of OEMs' new products in the long run, and re-licensing TPRs is a feasible strategy. The underlying hypothesis in these researches is that OEMs can either embrace the secondary market or try to eliminate it.

In the e-products industry, however, OEMs normally have little control over the secondary market. There is no software copyright problem for most electrical and electronic products, such as Android phones, so mandatory authorization for refurbishing is not required. TPRs decide if they are willing to accept OEMs' refurbishing authorization or not. Under this background, OEMs chooses whether and how to cooperate with the secondary market, instead of eliminating it. Based on the review of the literature, research on the refurbishing authorization in the e-products industry is still lacking, with the authorization strategy as a possible choice for both OEMs and TPRs. Our study attempts to fill the research gap. We focus on the refurbishing authorization strategy and study the conditions under which the authorization strategy is optimal for an OEM, in comparison with the strategy of refurbishing itself and the strategy of not engaging in refurbishing. We provide interesting managerial insights for OEMs who might be confused regarding the choice of refurbishing strategy.

The research motivation is as follows. Under what conditions should an OEM choose the authorization policy? Will the OEM and authorized TPR achieve a win-win outcome through authorization cooperation? Our paper will examine the following three strategies: 1) OEM does not involve in refurbishing work at all; 2) OEM engages in refurbishing itself; and 3) OEM provides TPR with refurbishing authorization. Once an OEM authorizes a TPR, there are different payment policies to choose (Kulatilaka and Lin, 2006). A commonly used option is the price-rate payment policy, where there is a charge for each piece of refurbished product sold (Oraiopoulos et al., 2012). Our paper will adopt this payment policy.

The remainder of the paper is organized as follows. In the next section, we provide a review of the literature. Section 3.1 builds a basic model with no involvement of the OEM in refurbishing. Section 3.2 explores the strategy of OEM engaging in refurbishing work. In Section 3.3, the refurbishing authorization model is studied. The conditions under which the OEM and TPR achieve a win-win outcome are examined. In Section 4, comparison of the three strategies is carried out using numerical simulations. Section 5 provides additional discussion and assumes that an OEM has a refurbishing cost advantage. In Sections 6 we discuss managerial implications. Finally, Section 7 presents concluding remarks.

2. Literature review

Remanufacturing has received growing attention in recent years. A stream of literature has focused on how OEMs manage their manufacturing and remanufacturing activities, such as Atasu et al. (2008a), Chen and Chang (2013), Francas and Minner (2009), Ferrer and Swaminathan (2010), Vadde et al. (2011) and Kenne et al. (2012). Due to the fast development of the secondary market, the literature on remanufacturing has concentrated primarily on the market segmentation between an OEM and a competing third party (Wu, 2013). Our paper draws on several streams of literature, each of which are reviewed below.

The difference of market recognition between new and remanufactured products has long been a concern for OEMs and remanufacturers. Some early studies assume that a remanufacturing process can be incorporated as a part of an OEM's original production system and there is no distinction between remanufactured and new products (Savaskan et al., 2004). Later research has considered a remanufactured product as a substitute for new products (Debo et al., 2006; Chen and Chang, 2013). Other studies focus on the consumers' value-discount or tolerance factor for a remanufactured product and assume that consumers show higher preference for new products than remanufactured products (Ferrer and Swaminathan, 2010; Oraiopoulos et al., 2012; Wu, 2013; Abbey et al., 2015a). Gan et al. (2017) prove that the customer acceptance towards remanufactured product will affect both the pricing decisions and profit of supply-chain members. More recent research has begun to empirically examine consumers' perceptions of remanufactured products. Abbey et al. (2015b) find that product quality is more important than discounting for remanufactured products. Several factors affecting consumers' purchase intention include quality knowledge, cost knowledge, and green knowledge (Wang and Hazen, 2016), energy saving, material saving and emission-reduction information (Wang et al., 2016), energy-efficient offerings (Khor and Hazen, 2017), and e-service offerings (Xu et al., 2017).
remanufacturers. In Majumder and Groenevelt (2001), an OEM exerts influence over remanufacturing by controlling the quantity of new products. In Wu (2013), an OEM can set the level to which products can be disassembled without force. Results show that this product-design strategy to be effective for the OEM in competing with a remanufacturer. Orsdemir et al. (2014) introduce quality of new products as an important factor and study how an OEM and independent remanufacturer competitively decides the production quantities. Ferguson and Toktay (2006) indicate that a third-party remanufacturer will cannibalize the market share of an OEM’s new products and suggest two entry-deterrent strategies for the OEM: remanufacturing and preemptive collection. Oraiopoulos et al. (2012) examine whether an OEM favor stimulating or discouraging the secondary market through charging a relicensing fee to the buyer of the refurbished product. Zou et al. (2016) compare two modes of outsourcing and authorization that OEMs could take to a third-party remanufacturer on branded or patented products.

Several studies investigate how OEMs deal with competing remanufacturers when OEMs are also engaging in refurbishing themselves. Ferrer and Swaminathan (2006) examine the competition between an OEM and a TPR in multiple periods, and find that when the competition intensifies, the OEM tends to remanufacture all collected products. Bulmus et al. (2014) examine the effect of competition between a remanufacturer and an OEM. They find that if the cost benefit of remanufacturing diminishes and the remanufacturer acquires more returned products, the OEM will manufacture less new products in the first period.

More recent studies find that the secondary market may bring some benefits to OEMs. Oraiopoulos et al. (2012) point out that the resale effect from the secondary market helps OEMs to increase sales of new products. Abbey et al. (2015a) find that an OEM can mitigate the effects of cannibalization from remanufactured products and increase its profitability after appropriate pricing of new products. Wu and Zhou (2016) prove that the entry of one or more remanufacturers may lead to a higher profit for the two competing OEMs. Through behavioral experiments, Agrawal et al. (2015) show that third-party remanufactured products have a positive effect on the perceived value of new products, which may actually be beneficial for an OEM. Accordingly, OEMs may need to reconsider the impact of the secondary market on their business.

Related literature with refurbishing authorization is still lacking. Oraiopoulos et al. (2012) and Zou et al. (2016) assume that relicensing/authorization is mandatory for remanufacturers, due to the issues of software copyright or patents. Examples include Cisco and Sun, which produce big servers and provide specialized software. Without the refurbishing authorization of OEMs, refurbished products cannot be sold at all. However, an active secondary market may not be controlled by OEMs (Xiong et al., 2016). For most e-products, mandatory authorization for refurbishing is not required. In reality, it is very common to find that third-party remanufacturers engage in refurbishing and sell refurbished products (Liu et al., 2016). For example, there are numerous TPRs selling refurbished products on eBay. TPRs can decide if they want to seek refurbishing authorization.

Related academic research on whether refurbishing authorization is a good choice for an OEM in the electrical and electronics industry is still limited. Our paper attempts to fill this research gap. We study the conditions under which the authorization strategy is optimal for an OEM, in comparison with the strategy of refurbishing itself and the strategy of not engaging in refurbishing. Specifically, we examine three scenarios:

1) OEM does not engage in refurbishing and there is an unauthorized TPR.
2) OEM engages in refurbishing in-house and there is an unauthorized TPR.
3) Refurbishing authorization model.

Each of these scenarios is discussed next.

3. Game model

3.1. Basic model with OEM’s not engaging in refurbishing

In the basic model, there is no refurbishing authorization and the OEM does not engage in refurbishing. The model consists of one OEM selling new products and one TPR collecting old products and selling refurbished products, as shown in Fig. 1. The competition between new and refurbished products is characterized in one production period. The OEM and TPR decide on the prices of new and refurbished products, respectively.

In reality, a tremendous amount of scrapped e-products are still stored by households. According to a survey on US households by Saphores et al. (2009), there are at least 4.1 small and 2.4 large e-products held in each household. Sabbaghi et al. (2015) find that commercial consumers have even stored computers more than household consumers. Sabbaghi et al. (2016) point out that it is empirically proven that consumers tend to keep the electronics in storage after they stopped using them. Therefore, the potential recyclable volume of e-products is huge. Meanwhile, the frequent updating and upgrading accelerates the growth of scrapped e-products. Furthermore, refurbishing quantity is only one small proportion of all waste products because refurbished products need to be “as-new” and will restrict the quality level of old products. Therefore, the paper assumes there is no upper limit for the total available refurbishing quantity.

Irrespective of prices, consumers have different preference degrees on new and refurbished products, and on authorized and non-authorized refurbishing channel. Assume that the consumers’ preference degree is 1 for new products, and α for refurbished products, with the condition that $0 < \alpha < 1$ (Abbey et al., 2015a; Oraiopoulos et al., 2012). Assume that the consumers’ preference degree is 1 for the authorized channel, and β for the non-authorized channel. Here $0 < \beta < 1$ (Ferrer and Swaminathan, 2006). α and β can be considered as the consumer perceived quality factors, which includes attributes such as warranty period, physical appearance and technical specification (Vorasayan and Ryan, 2006).

In order to distinguish these three strategies, the variables will use subscripts $i = b, m, a$ to represent the basic model without OEM’s engaging in refurbishing, the OEM’s refurbishing model, and refurbishing authorization model.

As such, the market recognition degrees on new products, authorized refurbished products, and non-authorized refurbished products are 1, α and αβ, respectively. In the basic model, we assume the OEM’s and TPR’s production costs on new and refurbished products to be $c_N$ and $c_R$, prices to be $P_N$ and $P_R$, and the corresponding sales quantities to be $q_N$ and $q_R$, respectively.

Suppose that consumers’ willingness to purchase $\theta$ has a uniform distribution, that is $\theta \sim U[0,1]$ (Ferrer and Swaminathan, 2006). If consumers buy new products, then the utility that they obtain is $U_N = \theta - P_N$. If consumers choose the authorized refurbished products, then the utility is $U_R = \alpha \theta - P_R$. If the non-authorized refurbished products are purchased, then the consumers’ utility is $U_R = \alpha \beta \theta - P_R$.

In the basic model, consumers only choose between the OEM’s new products and the TPR’s refurbished products. Here we define $\varphi = \alpha \beta$.

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**Fig. 1. Basic model without OEMs engaging in refurbishing.**
0 < \phi < 1. \phi \ refers to the degree of market recognition on non-authorized refurbished products. For any \theta, consumers will compare the utilities from purchasing new and refurbished products before making the purchasing decision. If \text{UN} \geq \text{UH}, that's \theta \geq \frac{P_N - P_R}{\text{cR}} P_N \ then new products will be purchased. If \text{UN} < \text{UH} & \text{UH} > 0, that's \frac{P_N}{\text{cR}} < \frac{P_N - P_R}{\text{cR}} \ then the consumers will choose to buy refurbished products. Therefore, for \theta \sim U[0, 1], the sales quantity of new products is 1 - \frac{P_N - P_R}{\phi P_R}, and that of refurbished products is \frac{P_N - P_R}{\phi P_R}.

\begin{equation}
q_N = 1 - \frac{P_N}{1 - \phi} + \frac{P_R}{1 - \phi}, \quad q_R = \frac{\phi}{\phi(1 - \phi)} P_N - \frac{1}{\phi(1 - \phi)} P_R
\end{equation}

The OEM and TPR decide on products prices at the same time. In the basic model, the OEM's profit is \(\pi_{N,b} = (P_N - c_R)q_N\) and TPR's profit is \(\pi_{R,b} = (P_R - c_R)q_R\). After substituting \(q_N\) and \(q_R\) into formula (1), the profit functions are concave with respect to \(P_N\) and \(P_R\). By combining the first-order derivatives with respect to these two decision variables, we obtain the only equilibrium outcome as follows.

\begin{equation}
P_{N,b}^* = \frac{2c_N + c_R + 2 - 2\phi}{4 - \phi} \quad \text{and} \quad P_{R,b}^* = \frac{2c_R + \phi c_N + (1 - \phi)}{4 - \phi}
\end{equation}

\begin{equation}
\begin{align*}
q_{N,b}^* &= \frac{2 - 2\phi - (2 - \phi) c_N + c_R}{4 - 5\phi + \alpha^2} \quad \text{and} \quad q_{R,b}^* = \frac{\phi (1 - \phi) + \phi c_N + c_R (2 - 2\phi)}{\phi (4 - \phi)(1 - \phi)}.
\end{align*}
\end{equation}

In the equilibrium, the OEM's profit is \(\pi_{N,b}^* = \frac{2 - 2\phi - (2 - \phi) c_N + c_R}{4 - 5\phi + \alpha^2} \phi (1 - \phi) + \phi c_N + c_R (2 - 2\phi)}{(4 - \phi)(1 - \phi)}\), and TPR's profit is \(\pi_{R,b}^* = \frac{\phi (1 - \phi)}{\phi (4 - \phi)(1 - \phi)}\).

Under the setting of positive sales quantities, there is a constraint \(q_{\text{low}} < \phi < q_{\text{high}}\) to meet. Here \(q_{\text{low}} = \frac{1}{2}(1 + \phi c_N + c_R - \sqrt{-8c_R + (1 + \phi c_N + c_R)^2})\) and \(q_{\text{high}} = \frac{2 - 2\phi - c_N}{2 - \phi}\) are the upper and lower limits. This constraint is used in static analysis of the equilibrium outcome with regards to \(\phi\).

Next, we define \(\phi_1 = 4 - \sqrt{6 - 2c_N - c_R}, \phi_2 = 2 - 2\phi - c_N - \sqrt{2c_N - 6 + 2c_N - c_R - c_R}\).

By comparison, we have \(\phi_2 > \phi_2\).

**Corollary 1.** *(See Proofs in Appendix.)*

(i) If \(c_L < \frac{2}{3} c_N\) and \(\phi \in [\phi_1, \phi_{\text{high}}]\), \(\frac{\partial q_{N,b}^*}{\partial \phi} < 0\); or else \(\frac{\partial q_{N,b}^*}{\partial \phi} > 0, \frac{\partial q_{R,b}^*}{\partial \phi} > 0\).

For the TPR, as the market recognition on non-authorized refurbished products \(\phi\) increases, its refurbishing quantity goes up, but its pricing strategy depends on the TPR's profitability. If the refurbishing cost \(c_R\) is low enough and \(\phi\) is quite high, indicating the TPR's competitiveness is strong, then the price of refurbished products is reduced to stimulate additional sales. If \(c_R\) is high enough, the TPR will raise price as \(\phi\) increases.

(ii) \(\frac{\partial q_{R,b}^*}{\partial \phi} < 0, \text{if} c_R > (6c_N - 2)/5\) and \(\phi \in [\phi_{\text{low}}, \phi_2]\), \(\frac{\partial q_{R,b}^*}{\partial \phi} > 0\); or else \(\frac{\partial q_{R,b}^*}{\partial \phi} < 0\).

For the OEM, as \(\phi\) increases, the price of new products drops, but the change in the sales quantity is dependent on \(c_R\) and \(\phi\). If \(c_R\) is high enough and \(\phi\) is quite low, indicating low TPR competitiveness, then the sales quantity of new products increases; otherwise, the sales quantity decreases. Let us examine the pricing of products in an OEM's official website and eBay. Confronted with the competition from TPRs who sell refurbished products on eBay, an OEM could decrease the price of new products slightly. Since the TPRs' refurbished products are still weak in competitiveness, the OEM's strategy of lowering the price will lead to a higher market demand. The outcome is that as \(\phi\) increases, we have an expanding market for the new products.

(iii) \(\frac{\partial q_{N,b}}{\partial \phi} \leq 0, \frac{\partial q_{R,b}}{\partial \phi} < 0\).

We note that a rise in consumers' preference for refurbished products will lead to a decrease in the OEM's profit and an increase in the TPR's profit.

**Proposition 1.** *(See proofs in Appendix.)*

There exists a threshold value \(\phi_3\) such that, if \(\phi \in (\phi_{\text{low}}, \phi_3]\), then \(q_{N,b}^* > q_{R,b}^*\); if \(\phi = \phi_3\), then \(q_{N,b}^* = q_{R,b}^*\).

When \(\phi\) is sufficiently high, the sales quantity of refurbished products will be even equal to that of refurbished products, which shows a strong market encroachment of refurbished products on new products.

### 3.2. OEM's refurbishing model

In the OEM's refurbishing model, a refurbishing dual-channel structure is formed. Since the OEM is now engaging in refurbishing work, it enjoys a brand advantage and its refurbished products have higher consumer preference.

Compared with those manufacturers who originally have no refurbishing processing experience, TPRs specialize in recycling and are usually equipped with professional disposing equipment. Therefore, the paper assumes that a TPR has a lower refurbishing cost than an OEM. The TPR's refurbishing cost is \(c_R\), and the OEM's cost is \(c_R + g\). Here \(g\) is assumed to be the OEM's extra refurbishing cost.

As shown in Fig. 2, the OEM collects old products and sells new and refurbished products in the market. The TPR collects old products and sells non-authorized refurbished products. Assume that the prices of new products, refurbished products and non-authorized refurbished products are \(P_N, P_A\) and \(P_R\) respectively. Using the method similar with that in the basic model, demand functions for the three kinds of products are obtained as follows.

\begin{align*}
q_N &= 1 - (P_N - P_A)/(1 - \alpha), q_A \\
&= (P_N - P_A)/(1 - \alpha) - (P_A - P_R)/(\alpha - \alpha\beta), q_R \\
&= (P_A - P_R)/(\alpha - \alpha\beta) - P_R/\alpha\beta
\end{align*}

The OEM's profit function is \(\pi_{N,a} = (P_N - c_N)q_N + (P_A - c_R - g)q_A\), and the TPR's profit function is \(\pi_{R,a} = (P_R - c_R)q_R\).

Substituting the demand functions into the above profit functions, and combining the first-order derivatives with respect to \(P_N, P_A\) and \(P_R\), we obtain the only equilibrium outcome as follows.

\begin{align*}
P_{N,a}^* &= \frac{4 + c_R(4 - \beta) - \beta + \beta g - 3\alpha\beta + c_R(2 + \beta) - 2(4 - \beta)}{4 - \beta} P_{N,a}^* \\
&= \frac{3\alpha + 2(\beta + \alpha - \alpha\beta)}{4 - \beta} P_{N,a}^* = \frac{c_R(2 + \beta) + \beta g + 3\alpha\beta - 2(4 - \beta)}{4 - \beta}
\end{align*}

In the equilibrium, for the OEM, the sales quantity of new products is

![Fig. 2. OEM's refurbishing model.](image-url)
Proposition 2. (See Proofs in Appendix)

(i) \[ \frac{\partial q_{\alpha \omega}}{\partial \alpha} < 0, \frac{\partial q_{\beta \omega}}{\partial \beta} > 0, \frac{\partial q_{P_{\alpha \omega}}}{\partial \alpha} > 0, \quad \text{if} \quad 2g + 3c_\omega > 6\alpha, \text{then} \quad \frac{\partial q_{P_{\beta \omega}}}{\partial \beta} > 0, \text{or else} \quad \frac{\partial q_{P_{\beta \omega}}}{\partial \beta} < 0. \] 

The correlation between \( \beta \) and the prices of OEM’s products depends on the refurbishing costs.

(ii) \[ \frac{\partial q_{\alpha \omega}}{\partial \alpha} < 0, \frac{\partial q_{\beta \omega}}{\partial \beta} > 0 \text{ if } 2g_\omega(1 - \beta) < g_\omega, \frac{\partial q_{\alpha \omega}}{\partial \alpha} < 0 \text{ or else } \frac{\partial q_{\beta \omega}}{\partial \beta} > 0. \text{ The TPR’s products sales will not necessarily go up as } \alpha \text{ increases.} \]

As the consumers’ preference for refurbished products \( \alpha \) rises, the sales quantity of new products will decrease and that of the OEM’s products will increase. However, the TPR’s products sales may increase. Results show that if the TPR’s refurbishing cost is low and the OEM’s extra cost is relatively high, then the TPR’s products sales is negatively correlated with \( \alpha \), which is different from the results in the basic model. In the OEM’s refurbishing model, the OEM sells refurbished products as well as new products. When the consumer preference for refurbished products \( \alpha \) increases, the refurbished products are more popular and will have higher market share. The secondary market becomes “a bigger cake” to both the OEM and TPR. However, for the new products, the sales quantity will drop and the OEM has to cut prices. Under this situation, the OEM will raise the price of refurbished products, in order to make the new products relatively attractive and protect the sales of new products, and also to earn the market bonus that a higher \( \alpha \) provides. Since the competitor adopts the strategy of raising prices, the TPR also increases its price, but by less than one half, \[ \frac{\partial q_{\beta \omega}}{\partial \beta} > 2. \]

Although TPR has increased the price of refurbished products, the sales quantity does not necessarily drop. Only when the OEM’s extra refurbishing cost is sufficiently high, the outcome is that the price of its refurbished products will increase to a high level. As a result, the TPR will also raise its price to obtain a bigger profit margin, which may discourage some sales.

Corollary 3. (See Proofs in Appendix)

(i) \[ \frac{\partial q_{\beta \omega}}{\partial \beta} > 0, \frac{\partial q_{\alpha \omega}}{\partial \alpha} < 0, \frac{\partial q_{\beta \omega}}{\partial \beta} < 0. \] 

An increase in the consumers’ preference degree on refurbished products may not be a good thing for the OEM. This is because the competitive pressure from the TPR’s refurbished products will be enhanced. Furthermore, as the difference in degree of market recognition between the OEM’s new and refurbished products becomes smaller, the advantage from price discrimination is reduced. If refurbishing cost is relatively low, then the cost advantage from refurbishing will cause the OEM to decrease the price of refurbished products and expand the sales volume. Thus, the OEM will expect a higher \( \alpha \).

3.3. Refurbishing authorization model

We develop a two-stage Stackelberg game. In the first stage, the OEM sets an authorization fee. The TPR chooses whether to seek authorization or not. In the second stage, the OEM and TPR decide on prices of the new products and refurbished products, respectively. We proceed backwards to derive the equilibrium.

We discuss the optimal authorization fee for the OEM and the acceptable authorization fee for the TPR. The TPR is willing to accept authorization only if the TPR’s profit does not drop after being authorized. Likewise, the OEM will only authorize the TPR if the OEM benefits from this arrangement. As such in the authorization strategy, if the profits of both the TPR and OEM increase, an authorization agreement will be reached. The outcome is a win-win strategy for the TPR and OEM.

In order to obtain the authorization qualification, a TPR needs to meet some requirements, such as improving production processes, investing in new technology and labor training. Therefore, compared with the model without authorization, the authorized TPR pays an extra cost \( C \). Here we assume \( C \) to be exogenous. In the authorization model, consumers choose between the new products and the authorized TPR’s refurbished products. Using the method similar to the basic model, demand functions for the two kinds of products are obtained as follows.

\[ q_N(T) = 1 - \frac{P_N}{1 - \alpha} + \frac{P_R}{1 - \alpha}, \quad q_R = \frac{\alpha}{a(1 - \alpha)}P_N - 1 \text{ (6)} \]

In the piece-rate payment policy, the authorized TPR pays the OEM an authorization fee for every piece of refurbished products sold. The OEM decides on the piece-rate authorization fee \( S \).

The OEM’s profit function is \( \pi_{OA} = (P_N - c_N)q_N + Sq_R \), and the authorized TPR’s profit function is \( \pi_{RA} = (P_R - c_R)q_R + S(1 - \alpha)q_R - C \).

Substituting demand functions (formula (6)) into the above profit functions, and combining the first-order derivatives with respect to \( P_N \) and \( P_R \), we obtain the unique equilibrium outcome as follows.

\[ P_{N,\omega} = \frac{2c_N + c_R + 3S - 2a}{4 - a}, \quad P_{R,\omega} = \frac{2c_R + (1 + c_R - a)S + S(2 + a)}{4 - a} \text{ (7)} \]

In the equilibrium, the sales quantity of new products is \( q_{N,\omega} = a(c + c_R - (2 + a) + (2 + a)S(1 - a)S) \), and the OEM’s profit is \( \pi_{OA} = (S^2 - c_N - (2 - a) + (2 + a)S(1 - a))a + 2c_N(1 - S) + 2(2c_R(1 - S) + 2(2 + a) - 2a)S + S(2 - a) + S^2(1 - a)S(a^2) \).

The sales quantity of refurbished products is \( q_{R,\omega} = \frac{(c + c_R - (2 + a) + (2 + a)S(1 - a))a + 2c_N(1 - S) + 2(2c_R(1 - S) + 2(2 + a) - 2a)S + S(2 - a) + S^2(1 - a)S(a^2)}{(4 - a)S - 1 - a} - C \).

Corollary 3. (i)

\[ \frac{\partial q_{\omega}}{\partial S} = 0, \text{ and } \frac{\partial q_{\omega}}{\partial \alpha} < 0. \]

As the piece-rate authorization fee \( S \) increases, both prices of new products and refurbished products go up, and the marginal effect on the price of new products is bigger. In addition, both sales quantities of new and refurbished products will drop, and the marginal effect on the sales quantity of refurbished products will be higher.
The condition for the TPR to accept authorization is that its profit does not decrease, meaning that \(\pi_{\alpha,\beta} - \pi'_{\alpha,\beta} \geq 0\), which needs \(S < \bar{S}\).

Here \(S = \frac{(-16(1 + c_R)\alpha\beta + \alpha^2\beta^2 - \alpha\beta)(9 + (2 + c_R)\beta) - c_N(2 - 3\alpha + \alpha^2)\beta(-4 + \alpha\beta)^2(-1 + \alpha\beta) + \alpha^2\beta^2(24 + 9(2 + c_R)\beta) + (1 + c_N)\beta^2}{\alpha^2\beta(16 + 24(2 + c_R)\beta + 9(1 + c_N)\beta^2 + 8\alpha\beta(4 + 3\beta) + c_N(2 + 3\beta))} + \sqrt{\frac{(-4 + \alpha)^2(-1 + \alpha)^3\beta(-4 + \alpha\beta)^2(-1 + \alpha\beta)(1 + c_N)\beta^2}{c_N(-2 + \alpha\beta)^2}(1 + \alpha\beta)^\beta(-4 + \alpha\beta)^2(-1 + \alpha\beta)^\beta} + \frac{\sqrt{\frac{c_N(-2 + \alpha\beta)^2}{c_N(1 + \alpha\beta)^\beta}}}{}\). Define \(S' = \frac{c_N(-2 + \alpha\beta)^2}{c_N(1 + \alpha\beta)^\beta}\). If \(S < S'\), then the OEM's profit is an increasing function of authorization fee; otherwise, it is a decreasing function.

Proposition 3. The OEM's profit is an inverted-U-shape curve with respect to authorization fee. As such a higher authorization fee is not always better for the OEM.

According to Proposition 3, a higher authorization fee does not always mean higher profit for the OEM. When the piece-rate authorization fee is increased, it appears that the OEM's profit would go up. However, when the piece-rate authorization fee is increased too much, the refurbishing product price will go up and demand will drop significantly. The final outcome is that the TPR's profitability becomes weaker and with reduced demand for refurbished products, the OEM's profit will also drop.

For the OEM, the optimal authorization fee is \(S'\). It is clear that \(S'\) is a decreasing function of \(c_R\) and \(c_N\), and an increasing function of \(c_R\). When the production costs are sufficiently high and the consumers' preference for refurbished products is sufficiently low, the OEM is willing to choose a low authorization fee to sustain the development of the TPR.

We evaluate the outcome for different parameter values, specifically when \(S' \geq S\), or \(S < S'\). Numerical simulations show that to achieve \(S' \geq S\), \(\alpha\) needs to be sufficiently high and \(c_R\) is sufficiently low. With \(\alpha\) as the horizontal axis, Fig. 3 shows an example of the numerical simulation. Here \(c_F = 0.5 \ldots c_K = 0.1, 0.3 \beta = 0.4, 0.9\).

Corollary 4. When \(\alpha\) is sufficiently high and \(c_R\) is sufficiently low, there exists \(\beta_1\) and \(\beta_2\) (\(0 < \beta_1, \beta_2 < 1\)), such that when \(\beta \in [\beta_1, \beta_2]\), \(S' \geq S\). (See proofs in the Appendix).

Proposition 4. (i) When \(S' < \bar{S}\), the optimal authorization fee for the two firms is \(S'\). With \(S = S'\), numerical simulations show that \(\pi'_{\alpha,\beta} > \pi'_{\alpha,\beta}\). Because the TPR also obtains a higher profit, the OEM and the TPR will achieve a win-win outcome.

(ii) When \(S' > \bar{S}\), the optimal authorization fee for the two firms is \(\bar{S}\). With \(S = S'\), the TPR will accept authorization, but the OEM's profit may decrease. So an authorization agreement may not necessarily be reached.

According to Corollary 4 and Proposition 4, if the consumers' preference degree on refurbished products \(\alpha\) is high enough, the TPR's cost of refurbished products \(c_R\) is sufficiently low, and the consumers' preference on non-authorized channel \(\beta\) is not very high, then the optimal authorization fee for the OEM is lower than the upper limit that the TPR could accept for authorization, and two firms will achieve a win-win outcome. As such, when the refurbished products are popular in the market and TPR has a cost advantage, there will be potential benefits for the OEM and TPR to cooperate with each other. By relying on the OEM's brand advantage in improving consumers' preference on refurbished products, authorization will lead to market expansion, from which both the OEM and TPR will benefit.

4. Conditions for choosing refurbishing authorization

Based on the above results, we study the conditions under which the authorization strategy is optimal for the OEM. This section provides a comprehensive comparison of the OEM's profits under any value combination of parameters \((\alpha, \beta, c_N, c_R, g)\). Note that, we only study the scenario with positive demand for the TPR and the OEM, so there are multiple constraint conditions for the equilibrium in each model. We adopt Orapioulos et al. (2012) approach of only examining the conditions for the optimal strategy numerically. Proposition 7 provides the observation on profit comparison of the three strategies. Figs. 4 and 5 show a few examples with different parameter combinations.

Proposition 7. If the consumers' preference on refurbished products \(\alpha\) is not large enough, then refurbishing authorization is the optimal strategy; otherwise, the OEM should choose the other two strategies of refurbishing by itself or not engaging in refurbishing. The choice between the other two depends on the OEM's refurbishing cost \(c_R + g\) and the consumers' preference degree on non-authorized channel \(\beta\). If \(c_R + g\) and \(\beta\) are sufficiently high, the OEM should choose not to engage in refurbishing, or else, refurbishing by itself is better.

Fig. 4 plots the OEM's profits for the three strategies when \(\alpha\) is large \((\alpha = 0.9)\) and medium \((\alpha = 0.6)\), respectively. Here \(c_F = 0.35, c_R = 0.15\) and \(g = 0.1\). When \(\alpha = 0.9\), the constraint conditions for positive demand cannot be met with the refurbishing authorization strategy, so the
OEM could choose between refurbishing by itself and not engaging in refurbishing. When $\alpha = 0.6$, it is clear an authorization strategy is always dominant. When $\alpha = 0.9$, Fig. 5 plots the OEM's profits for the two strategies of refurbishing by itself and not engaging in refurbishing. The settings are $c_N = 0.35$, $c_R = 0.15$, $g = 0.05$. When $g = 0.05$, $\pi^*_{N,a} > \pi^*_{N,b}$, a refurbishing strategy is always better. When $g$ goes up from 0.05 to 0.15, the OEM's profit $\pi^*_{N,b}$ with the strategy of not engaging in refurbishing is not affected, but the OEM's profit $\pi^*_{N,a}$ using the strategy of refurbishing by itself decreases. With $g = 0.15$, it is clear that if $c_R = 0.1$, we have $\pi^*_{N,b} > \pi^*_{N,a}$ only when $\beta$ is very large (beyond 0.8); if $c_R = 0.15$, we have $\pi^*_{N,b} > \pi^*_{N,a}$ when $\beta$ is quite large (beyond 0.6). In summary, when $c_R + g$ and $\beta$ are large enough, not engaging in refurbishing is a better choice.

According to the above results, when the degree of consumers' preference on refurbished products is not very large, refurbishing authorization is the optimal strategy. Once an authorization agreement is reached, we have a win-win outcome for the OEM and the TPR. When the consumers' preference degree of refurbishing products is large enough, indicating that price is almost the only factor that influences the consumers' purchase decision, the OEM is not willing to provide authorization to the TPR because the TPR will be a stronger competitor after being authorized. Thus, the strategy of refurbishing by itself or not engaging in refurbishing is more suitable.

5. Further discussion on refurbishing cost

In the OEM's refurbishing model, we assume that the TPR's refurbishing cost is $c_R$, and the OEM's cost is $c_R + g$. Here $g$ is assumed to be the OEM's extra refurbishing cost. Now we relax the assumption and consider the situation that the OEM has the cost advantage of refurbishing, i.e. $g < 0$. Compared with the TPR, the OEM has both the advantages of higher consumer preference and lower refurbishing cost. The OEM will be a very strong competitor for the TPR. Therefore, once the OEM chooses to refurbish by itself, it will have more market share than the TPR in the secondary market.

With the new assumption, the OEM's choice will be influenced between the three strategies, namely, (1) not engaging in refurbishing, (2) refurbishing in-house, and (3) providing refurbishing authorization to TPRs. The strategy of OEM's refurbishing in-house will become more attractive due to the cost advantage of refurbishing. Next, we reexamine the conditions for choosing refurbishing authorization in Section 4.

Fig. 6 plots the OEM's profits for the three strategies when $\alpha$ is large.
The settings are dominant, as compared with the results shown in Fig. 4. If the consumers’ preference degree on non-authorized channel \( \beta \) is low, which means a big difference between authorized and non-authorized channels, then the OEM is more willing to refurbish by itself, due to a much higher competitiveness than the TPR in the secondary market. When the consumers’ preference degree on non-authorized channel \( \beta \) is high enough, the authorization strategy will be optimal.

With \( \alpha = 0.75 \), Fig. 7 plots the OEM’s profits for the two strategies of refurbishing by itself and not engaging in refurbishing. The settings are \( c_N = 0.35, c_R = 0.2 \) and \( 0.15, g = -0.01 \) and \(-0.05 \). When \( g = -0.05 \), \( \pi_{N,a} > \pi_{N,b} \), a refurbishing strategy is always better. When \( g \) goes up from \(-0.05 \) to \(-0.01 \), the OEM’s profit \( \pi_{N,a} \) with the strategy of not engaging in refurbishing is not affected, but the OEM’s profit \( \pi_{N,a} \) using the strategy of refurbishing by itself decreases. With \( g = -0.01 \), it is clear that we have \( \pi_{N,a} > \pi_{N,b} \) only when \( \beta \) is very large. In summary, when \( c_R + g \) and \( \beta \) are large enough, not engaging in refurbishing is a better choice.

With the new assumption of the OEM’s cost advantage in refurbishing, the condition for choosing the refurbishing strategy becomes more restrictive. If the consumers’ preference to refurbished products \( \alpha \) is not large enough, the strategies of refurbishing authorization and OEM’s refurbishing in-house should be considered. In addition, when the consumers’ preference for the non-authorized channel \( \beta \) is quite high, refurbishing authorization is the optimal strategy. This is because the lower refurbishing cost has already provided an advantage for the OEM to refurbish products in-house. Only when the advantage of high consumer preference to OEM’s or authorized channel is not so obvious, will the strategy of refurbishing in-house lose its attractiveness and then the authorization strategy will become a feasible choice.

6. Managerial implications

The results of our study provide managerial guidance for OEMs in the remanufacturing industry. The refurbishing authorization model shows the OEM’s profit with respect to authorization fee to be an inverted-U-shape curve. This indicates that a higher authorization fee does not always bring a higher profit for OEM. In reality, once OEMs choose to authorize TPRs in product refurbishing, the authorization fees should not be set too high. Excessive authorization fees may lead to a lose-lose outcome, even a failure of authorization cooperation between OEMs and TPRs. If a TPR has been a strong competitor and will be much stronger after authorization, the optimal authorization fee that OEM charges should be lower than the TPR’s participating threshold. In this scenario, they will achieve a win-win authorization cooperation strategy.

In the choice of refurbishing strategy, our findings also provide some managerial insights. In practice, some OEMs’ refurbishing behaviors are consistent with our model results. For example, HP and Lenovo compete in a fast-changing technology environment, and as a consequence refurbished products only have a low market recognition compared with new products. Furthermore, their recycling channel is still in its infancy and the acquisition quantity is limited, which makes the recycling cost relatively high. Therefore, providing authorization is the optimal strategy. However, for Apple, the consumer preference for its refurbished products is still high. It’s noticeable that the sales price difference between refurbished and new products is low on Apple website, while the sales price difference between authorized and non-authorized refurbished products is large in the market. The refurbishing channel makes a remarkable difference on sales prices. Thus, refurbishing by itself is better for Apple. Another example is Dell, a major PC manufacturer. Dell’s major advantages lie in low recycling cost and large acquisition quantity. Its recycling channel is the most mature, with convenient facilities for consumers almost all over the world. A large part of its consumer groups comes from businesses, institutes and governments, which contribute a large number of used products still in good quality. In addition, consumer preference for Dell’s refurbished products is quite high due to its brand environmental image. Therefore, Dell does most of its own refurbishing work.

7. Conclusion

In the electrical and electronics industry, TPRs selling refurbished products provide competition to OEMs selling new products. To address this threat, OEMs can use one of three strategies: (1) not engaging in refurbishing, (2) engages in refurbishing in-house, and (3) providing refurbishing authorization to TPRs. For TPRs, they can choose whether to be authorized by OEMs or not. Once authorized, the TPR’s refurbished products will have a higher market recognition. Our paper builds Bertrand competition models between an OEM and a TPR to study these strategies. In the equilibrium, the paper examines under what condition the refurbishing authorization strategy is optimal for the OEM and whether the OEM and the authorized TPR can achieve a win-win outcome.

Our research finds that when the OEM is not engaged in refurbishing work, the sales quantity of new products will be even lower than that of

\[ (\alpha = 0.75) \text{ and medium } (\alpha = 0.6), \text{ respectively. Here } c_N = 0.35, c_R = 0.15 \text{ and } g = -0.01. \text{ When } \alpha = 0.75, \text{ the constraint conditions for positive demand cannot be met with the refurbishing authorization strategy, so the OEM could choose between refurbishing by itself and not engaging in refurbishing. When } \alpha = 0.6, \text{ the authorization strategy is not always dominant, as compared with the results shown in Fig. 4. If the consumers' preference degree on non-authorized channel } \beta \text{ is low, which means a big difference between authorized and non-authorized channels, then the OEM is more willing to refurbish by itself, due to a much higher competitiveness than the TPR in the secondary market. When the consumers' preference degree on non-authorized channel } \beta \text{ is high enough, the authorization strategy will be optimal.} \]

\[ \text{With } \alpha = 0.75, \text{ Fig. 7 plots the OEM's profits for the two strategies of refurbishing by itself and not engaging in refurbishing. The settings are } c_N = 0.35, c_R = 0.2 \text{ and } 0.15, g = -0.01 \text{ and } -0.05. \text{ When } g = -0.05, \pi_{N,a} > \pi_{N,b}, \text{ a refurbishing strategy is always better. When } g \text{ goes up from } -0.05 \text{ to } -0.01, \text{ the OEM's profit } \pi_{N,a} \text{ with the strategy of not engaging in refurbishing is not affected, but the OEM's profit } \pi_{N,a} \text{ using the strategy of refurbishing by itself decreases. With } g = -0.01, \text{ it is clear that we have } \pi_{N,a} > \pi_{N,b} \text{ only when } \beta \text{ is very large. In summary, when } c_R + g \text{ and } \beta \text{ are large enough, not engaging in refurbishing is a better choice.} \]

\[ \text{With the new assumption of the OEM's cost advantage in refurbishing, the condition for choosing the refurbishing strategy becomes more restrictive. If the consumers' preference to refurbished products } \alpha \text{ is not large enough, the strategies of refurbishing authorization and OEM's refurbishing in-house should be considered. In addition, when the consumers' preference for the non-authorized channel } \beta \text{ is quite high, refurbishing authorization is the optimal strategy. This is because the lower refurbishing cost has already provided an advantage for the OEM to refurbish products in-house. Only when the advantage of high consumer preference to OEM's or authorized channel is not so obvious, will the strategy of refurbishing in-house lose its attractiveness and then the authorization strategy will become a feasible choice.} \]

\[ \text{6. Managerial implications} \]

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\[ \text{In the choice of refurbishing strategy, our findings also provide some managerial insights. In practice, some OEMs' refurbishing behaviors are consistent with our model results. For example, HP and Lenovo compete in a fast-changing technology environment, and as a consequence refurbished products only have a low market recognition compared with new products. Furthermore, their recycling channel is still in its infancy and the acquisition quantity is limited, which makes the recycling cost relatively high. Therefore, providing authorization is the optimal strategy. However, for Apple, the consumer preference for its refurbished products is still high. It's noticeable that the sales price difference between refurbished and new products is low on Apple website, while the sales price difference between authorized and non-authorized refurbished products is large in the market. The refurbishing channel makes a remarkable difference on sales prices. Thus, refurbishing by itself is better for Apple. Another example is Dell, a major PC manufacturer. Dell's major advantages lie in low recycling cost and large acquisition quantity. Its recycling channel is the most mature, with convenient facilities for consumers almost all over the world. A large part of its consumer groups comes from businesses, institutes and governments, which contribute a large number of used products still in good quality. In addition, consumer preference for Dell's refurbished products is quite high due to its brand environmental image. Therefore, Dell does most of its own refurbishing work.} \]

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\[ \text{Our research finds that when the OEM is not engaged in refurbishing work, the sales quantity of new products will be even lower than that of} \]

\[ \text{Fig. 6. OEMs profits for different strategies } (c_N = 0.35, c_R = 0.15, g = -0.01). \]
refurbished products when the degree of consumer preference on refurbished products is high enough. This result indicates there is market encroachment of refurbished products on new products. When the OEM is engaging in refurbishing in-house, an increase in the degree of consumer preference for refurbished products will enhance the TPR’s profit, but not necessarily improve the OEM’s. The OEM benefits only if there is a sufficient cost disadvantage from refurbishing.

For the strategy of refurbishing authorization, the OEM’s profit is an inverted-U-shape curve with respect to an authorization fee. A higher authorization fee is not always better for the OEM. If the piece-rate authorization fee is too high, the TPR’s profitability becomes weaker and the refurbishing volume will drop significantly, which eventually will lead to a decrease in the OEM’s profit. It is interesting to note that if the TPR has been a strong competitor before refurbishing authorization, then the optimal authorization fee for the OEM may be lower than the upper limit that the TPR would be willing to accept, and a win-win outcome could be realized.

For the three possible strategies, our paper examines the condition under which the authorization strategy is the best choice for the OEM. When the consumers’ preference on refurbished products is not large enough, a refurbishing authorization strategy is dominant. Once the authorization agreement is reached, the authorized TPR represents an end products to the OEM’s product line, which will result in market expansion and price discrimination. When the consumers’ preference on refurbished products is large enough, the other two strategies are better for the OEM. If the OEM’s refurbishing cost is relatively high and the consumers’ preference on refurbished channel is high, then refurbishing is not attractive for the OEM and there is no need to take the risk of engaging in refurbishing work. Otherwise, the OEM should engage in refurbishing in-house.

In recent years the trend in sustainability has resulted in an increasing secondary market for refurbished products in the electric and electronics industry. Third part resellers or remanufacturers are refurbishing electronic products with or without authorization from the original equipment manufacturers. As a result, manufacturers have to decide how to react to the competition that refurbished products present to new product sales. Authorizing third party resellers has become increasingly popular; however, there are still many firms who are confused about this strategy. Our results provide valuable managerial insights for manufacturers interested in the refurbishing business.

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Appendix

Corollary 1. Proof: \( \frac{\partial \pi^2}{\partial \bar{t}} = \frac{4 + 4C_N + 2C_R}{(4 - \bar{q})^2} \), and its plus-minus sign depends on \( 4 + 4C_N + 2C_R - 8 \bar{q} < 0 \). Since \( 0 \leq \bar{q} \leq 1 \), it is not difficult to obtain that when \( 0 \leq \bar{q} \leq 1 \), it is not difficult to obtain that when \( 4C_N + 2C_R > 3 \), \( \frac{\partial \pi^2}{\partial \bar{t}} > 0 \).

When \( 4C_N + 2C_R < 3 \), if \( \bar{q} \in [\varphi_{\text{low}} - \sqrt{2} \sqrt{6 - 2C_N - C_R}] \), then \( \frac{\partial \pi^2}{\partial \bar{t}} > 0 \); and if \( \bar{q} \in [4 - \sqrt{2} \sqrt{6 - 2C_N - C_R}], then \frac{\partial \pi^2}{\partial \bar{t}} > 0 \).

Numerical simulation results show that as all parameters change, \( \frac{\partial \pi^2}{\partial \bar{t}} \) is still positive.

Here, we define \( \varphi_{\text{high}} = 4 - \sqrt{2} \sqrt{6 - 2C_N - C_R} \). 

(ii) \( \frac{\partial \pi^2}{\partial \bar{q}} = \frac{(6 - 2C_N - C_R)}{2(4 - \bar{q})^2} \), and its plus-minus sign depends on \( T = C_N - 2(1 + \bar{q})^2 - C_R (6 - 4 \bar{q} + \bar{q}^2) \). \( T \) is a decreasing function with \( \bar{q} \) in the interval \( 0 \leq \bar{q} \leq 1 \).
regard to \( \varphi \in [\varphi_{low}, \varphi_{high}] \).

When \( 2 + 5c_R - 6c_N < 0, \frac{\partial^2 \varphi}{\partial \varphi^2} < 0. \)

When \( 2 + 5c_R - 6c_N > 0, \) if \( \varphi \in \left[ \varphi_{low}, \frac{2-2c_N-c_R-\sqrt{2c_N^2+(-6c_R^2+4c_N+4c_R+4c_R^2)\varphi}}{2-c_N} \right], \) then \( \frac{\partial^2 \varphi}{\partial \varphi^2} > 0; \) and if \( \varphi \in \left[ \frac{2-2c_N-c_R-\sqrt{2c_N^2+(-6c_R^2+4c_N+4c_R+4c_R^2)\varphi}}{2-c_N}, \varphi_{high} \right], \)

then \( \frac{\partial^2 \varphi}{\partial \varphi^2} < 0. \)

Here, we define \( \varphi_2 = \frac{2-2c_N-c_R-\sqrt{2c_N^2+(-6c_R^2+4c_N+4c_R+4c_R^2)}}{2-c_N}. \)

(iii) \( \frac{\partial^2 \varphi}{\partial \varphi^2} = \frac{(\varphi(1+c_R-\varphi)+c_R(-2+\varphi))(c_R(-8+18\varphi-9\varphi^2+2\varphi^3)+\varphi(-4+11\varphi-7\varphi^2+c_R(-4+\varphi+2\varphi^2)))}{(-4+\varphi)(1-\varphi)^2}. \)

Limited by the calculation complexity due to changing parameters, we analyse the plus-minus signs of \( \frac{\partial^2 \varphi}{\partial \varphi^2} \) by using numerical simulation. Results show that as all parameters change, in the constraints of positive sales quantities we have \( \frac{\partial^2 \varphi}{\partial \varphi^2} < 0 \) and \( \frac{\partial^2 \varphi}{\partial \varphi^2} > 0. \)

Proposition 1. 

Proof: \( \varphi_{N, \beta}' - \varphi_{B, \beta}' = \frac{2+g(1+c_R-\varphi)(3-\varphi)}{\varphi+\varphi(1+c_R-\varphi)} > 0. \)

It is easy to prove that when \( \varphi \in \left[ \varphi_{low}, \frac{1-3c_N+\sqrt{1-6c_R^2+8c_N+8c_R^2}}{2-c_N} \right], \) \( \varphi'_{N, \beta} > \varphi'_{B, \beta} \), and when \( \varphi \in \left[ \frac{1-3c_N+\sqrt{1-6c_R^2+8c_N+8c_R^2}}{2-c_N}, \varphi_{high} \right], \) \( \varphi'_{N, \beta} < \varphi'_{B, \beta} \).

Here, we define \( \varphi_3 = \frac{1-3c_N+\sqrt{1-6c_R^2+8c_N+8c_R^2}}{2-c_N}. \)

Corollary 2. 

Proof: (i) \( \frac{\partial^2 \varphi}{\partial \beta^2} = -\frac{c_R}{(1-\alpha)^2} < 0. \)

When \( 3c_R+2g > 6\alpha, \) \( \frac{\partial^2 \varphi}{\partial \beta^2} = \frac{3c_R-2g-6\alpha}{(4-\beta)(4-\beta)} \) if \( \frac{\partial^2 \varphi}{\partial \beta^2} > 0; \) when \( 3c_R+2g < 6\alpha, \frac{\partial^2 \varphi}{\partial \beta^2} = \frac{3c_R+2g-6\alpha}{(4-\beta)(4-\beta)} \).

\( \frac{\partial^2 \varphi}{\partial \beta^2} = \frac{6c_R+4g+(4-\beta)\varphi}{(4-\beta)^2} > 0. \)

\( \frac{\partial^2 \varphi}{\partial \alpha \partial \beta} = \frac{-c_R+c_N+g}{2(1-\alpha)^2} < 0. \)

(ii) \( \frac{\partial^2 \varphi}{\partial \alpha \partial \beta} = \frac{2c_R+4g-4c_R\alpha-8g\varphi+4c_R\varphi^2-2c_R\alpha^2+(-2c_R-2g+4c_R\alpha+4g-5c_N\alpha^2+3c_R\alpha^2+3g\varphi^2)\beta+(c_N\alpha^2-c_R\alpha^2-g\varphi^2)^2}{(2-\alpha)^2(4-\beta)(1-\beta)} \).

The monotonicity of \( \varphi'_{A, \beta} \) with respect to \( \alpha \) is decided by \( T = 2c_R + 4g - 4c_R\alpha + 8g\varphi - 4c_R\varphi^2 + 2c_R\alpha^2 + (2c_R-2g+4c_R\alpha+4g-5c_N\alpha^2+3c_R\alpha^2+3g\varphi^2)\beta+(c_N\alpha^2-c_R\alpha^2-g\varphi^2)^2 \).

The symmetry axis of \( T \) is beyond 1, \( -((2c_R-2g+4c_R\alpha+4g-5c_N\alpha^2+3c_R\alpha^2+3g\varphi^2)/(2(c_N\alpha^2-c_R\alpha^2-g\varphi^2))) > 1. \)

At the point \( \beta = 0, T = 2c_R + 4g - (4-4g-8g\alpha+4c_R\varphi)(c_R-2c_R-2g+4c_R\alpha+4g-5c_N\alpha^2+3c_R\alpha^2+3g\varphi^2) \) which is positive for \( \alpha \in [0, 1]. \) As such, when \( \beta = 0, T > 0. \)

At the point \( \beta = 1, T = 2g(1-\alpha)^2 > 0. \)

Therefore, we get \( T > 0, \) which means \( \frac{\partial^2 \varphi}{\partial \alpha \partial \beta} < 0. \)

\( \frac{\partial^2 \varphi}{\partial \alpha \partial \beta} = \frac{-2c_R+g(1-\alpha)(1-\alpha)(1-\beta)(2c_R+g(1-\alpha)(1-\alpha)(1-\beta))}{(-\alpha^2(4-\beta)(1-\beta))} > 0. \)

Proposition 2. 

Proof: \( \frac{\partial^2 \varphi}{\partial \alpha^2} > 0 \) when \( 2c_R(1-\beta) > \beta g, \frac{\partial^2 \varphi}{\partial \alpha^2} > 0; \) and when \( 2c_R(1-\beta) < \beta g, \frac{\partial^2 \varphi}{\partial \alpha^2} < 0. \)

\( \frac{\partial^2 \varphi}{\partial \alpha^2} = \frac{-2c_R\alpha^2(-4+\beta)^2(-1+\beta)-\alpha^2(-1+\beta)(-c_R^2(-4+\beta)^2+(1+\beta)(8+\beta))+(4(2-\beta)^2-8\alpha(2+\beta)^2)+\alpha^2(8-4\beta+\beta^2)+c_R(1-\alpha)^2(4(1-1+\beta)-8\alpha(-1+\beta)+\alpha^2(12-4\beta+\beta^2))}{2(1+\alpha)^2(4-\beta)(1-\beta)}. \)

The denominator is positive and the numerator \( -c_R\alpha^2(-4+\beta)^2+g(4(2-\beta)^2-8\alpha(-1+\beta)+\alpha^2(12-4\beta+\beta^2)) \) is a quadratic function of \( \alpha. \) Its highest value is equal to \( \frac{-c_R\alpha^2-c_R(4\beta+2\alpha-12\beta+\beta^2)(1+\alpha)^2(4-\beta)(1-\beta)}{2(1+\alpha)^2(4-\beta)(1-\beta)}, \) which is negative.

So \( \frac{\partial^2 \varphi}{\partial \alpha^2} < 0, \) \( \frac{\partial^2 \varphi}{\partial \alpha^2} \) is negatively correlated with \( c_R. \)
\[
\frac{d\bar{\pi}_R}{d\alpha} = \frac{-c_N\alpha^2(-4 + \beta)^2(-1 + \beta) + g(4(-2 + \beta)^2 - 8\alpha(-2 + \beta)^2 + a^2\beta(8 - 5\beta + \beta^2))}{-2(1-\alpha^2)\alpha^2(4(1-\beta))}.
\]

The denominator is negative and the numerator
\[-c_N\alpha^2(-4 + \beta)^2(-1 + \beta) + g(4(-2 + \beta)^2 - 8\alpha(-2 + \beta)^2 + a^2\beta(8 - 5\beta + \beta^2))\]
which is positive.

So \(\frac{d\bar{\pi}_R}{d\alpha}\) is negatively correlated with \(g\).

To sum up, \(\frac{d\bar{\pi}_R}{d\alpha}\) is positively correlated with \(c_N\), and negatively correlated with \(g\) and \(c_R\). Therefore, if refurbishing costs \(c_R\) and \(g\) are low, and new products cost \(c_N\) is relatively high, then \(\frac{d\bar{\pi}_R}{d\alpha} > 0\), or else \(\frac{d\bar{\pi}_R}{d\alpha} < 0\).

**Corollary 3. Proof:**

\[\frac{d\bar{\pi}_R}{d\alpha} = \frac{4(4(-2 + \beta)^2 - 8\alpha(-2 + \beta)^2 + a^2\beta(8 - 5\beta + \beta^2))}{-2(1-\alpha^2)\alpha^2(4(1-\beta))}\]

It is easy to determine that \(\frac{d\bar{\pi}_R}{d\alpha} < 0\). \(\bar{\pi}_{R,a} - \bar{\pi}_{R,b}\) is a decreasing function of \(S\). By solving the quadratic equation \(\pi^{*}_{R,a} = \pi^{*}_{R,b} = 0\), we can obtain a positive solution \(S\). Therefore, \(S < S^{*}\).

**Corollary 4. Proof:**

\[\frac{d(S - S^{*})}{d\beta} = \frac{(4\beta(-4 + \alpha\beta)(-1 + \alpha\beta))(6 + 9\alpha^2 - 4\alpha(4 + 11\alpha))}{16\sqrt{2\beta}(2 - 7\alpha\beta)}\]

When \(\beta < \frac{3\alpha^2 - 4\alpha - 1}{4\alpha - 1}\) and \(\beta > \frac{3\alpha^2 - 4\alpha + 1}{4\alpha - 1}\), \(\frac{d(S - S^{*})}{d\beta} < 0\). Therefore, at the point \(\beta = \min\left[\frac{3\alpha^2 - 4\alpha + 1}{4\alpha - 1}, \frac{3\alpha^2 - 4\alpha - 1}{4\alpha - 1}\right]\), the highest is the highest.

When \(\beta = 1, S = S^{*} < 0\). As such, when \(\alpha\) is sufficiently high, such that \(\frac{3\alpha^2 - 4\alpha - 1}{4\alpha^2 - 1} < 1\), then there exists \(\beta_1\) and \(\beta_2\) such that \(0 < \beta_1 < \beta_2 < 1\), when \(\beta \in [\beta_1, \beta_2]\), and \(S > S^{*}\).


