# A Theory of Organizational Dynamics: Internal Politics and Efficiency 

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#### Abstract

We consider a three-member organization in which one member retires in each period and the incumbent members vote to admit a candidate to fill the vacancy. Candidates differ in quality and belong to one of two types, and majority-type members share the total rent of that period. We characterize the symmetric Markov equilibria with undominated strategies and compare the long-term welfare among them. Unanimity voting is better than majority voting at promoting long-term welfare. In addition, organizations with a certain degree of incongruity perform better in the long run than either harmonious or very divided organizations. (JEL D23, D71, D72)


The long-term health and survival of an organization depend crucially on its ability to attract high caliber new members. However, internal politics, whereby different groups of incumbent members vie for control over the decision-making power of the organization, often interferes with admission of new members. ${ }^{1}$ In the process of admitting a candidate into an organization, incumbent members will look not only at his qualifications, but also at how his admission affects the future power structure of the organization. In this paper, we develop an infinite-horizon dynamic model to study how internal politics affect an organization's admission of new members. We also investigate how the dynamic interactions between internal politics and admission of new members affect the organization's long-term welfare.

We consider a three-member organization (club) in which one of the incumbent members is chosen randomly to exit in each period and, before knowing who will exit, the incumbent members vote to admit a candidate to fill the vacancy. Each player has two characteristics: quality and type. The uniformly-drawn quality represents a player's skills, prestige, or resources, which are valuable to every member

[^0]of the organization. However, internal politics are often anchored on things other than quality, such as race, gender, ideology, or personality. While the political structure of many organizations is often quite complicated, for simplicity we suppose that every player belongs to one of two types: left or right. Type matters in the sense that there is a fixed amount of rent (e.g., research funds, perks, prestigious positions) in each period and the majority type controls the rent allocation and distributes it equally among members of its type.

Because the amount of rent is fixed, the club's per period welfare is simply the sum of the quality of the club members minus the total search costs, and thus is independent of its power structure. In the first best solution, the founder or social planner of the club optimally trades off the search costs and the benefits of setting a high standard. In another benchmark, we suppose that there is no rent to grab and hence no internal politics in the club, so all incumbent members have identical preferences and will choose the same admission standard for both types of candidates. In this case, the equilibrium (called the "harmonious equilibrium") admission policy is inefficient because there is an "intertemporal free-riding" problem in that incumbent members do not take into account the effects of their admission decisions on future generations of club members. Consequently, all incumbent members set inefficiently low admission standards and search less relative to the efficient level.

In the presence of internal politics, we focus on symmetric Markov equilibria with undominated strategies in which incumbent members' strategies depend only on the current period type profile (state) of the club. We solve the most efficient equilibrium in terms of long-term welfare under both majority and unanimity voting rules in selecting new members. Under either voting rule, the solution crucially depends on the value of a variable interpreted as the degree of incongruity of the club. This variable is a function of the model's primitive parameters. It is smaller (the club is more congruous) when the available rent (and therefore the potential gain from internal politics) is smaller, or when the uncertainty over candidate quality is greater (so searching for good candidates is more important), or when the delay cost is higher (so the cost of internal politics is greater).

It turns out that the most efficient equilibria can be divided into two categories: "power-switching" equilibria, in which both types of candidates are admitted with positive probabilities in every state so power switches back and forth between the two types over time; and "glass-ceiling" equilibria, in which candidates of the minority type are never admitted in contentious states (when both types of incumbents are present), and hence the club will never experience power switches. Not surprisingly, under either voting rule, "power-switching" equilibria arise when the club is relatively congruous while "glass-ceiling" equilibria arise when the club is relatively incongruous. The most efficient equilibria under the two voting rules are different in the following two scenarios. In one scenario, when the degree of incongruity is intermediate, the only equilibrium under majority voting is glass-ceiling while unanimity rule still allows a power-switching equilibrium. This is because majority voting is more exclusive than unanimity voting, as under majority rule the majority-type incumbents can easily set a glass ceiling for the opposite type and keep control of the organization forever, but doing so is very costly under unanimity rule. In the other scenario, when the degree of incongruity is sufficiently low, the
power-switching equilibria under these two voting rules are different. Under majority rule, candidates of the minority type are discriminated against and face higher admission standards than those of the majority type in contentious states. Such an equilibrium is labeled "pro-majority power-switching." However, under unanimity rule, the opposite happens: candidates of the majority type are discriminated against in contentious states. This is because the majority-type incumbents have weaker incentives to fight with the minority-type incumbent so the minority-type incumbent can insist on admitting his favored candidate. Such an equilibrium is labeled "pro-minority power-switching."

Comparison of organization welfare for long-term equilibrium outcomes yields two main findings. First, the long-term welfare under unanimity voting is always greater than or equal to that under majority voting. Unanimity voting outperforms majority voting in the two scenarios described above. In the first scenario, unanimity rule still allows a power-switching equilibrium, but under majority rule only the less efficient glass-ceiling equilibrium exists. In the second scenario, the pro-minority power-switching equilibrium under unanimity rule achieves greater long-term welfare than the pro-majority power-switching equilibrium under majority rule. In the latter equilibrium, the majority-type incumbents set admission standards for their own type low enough to keep control of the club, exacerbating the intertemporal free-riding problem; while in the former equilibrium, both types of candidates face admission standards that are quite stringent, which helps overcome the intertemporal free-riding problem and improves long-term welfare. Second, when the club is relatively congruous, unanimity voting allows both types of incumbent members in a contentious state to raise the admission standards for candidates of the opposite type, but the admission standards are not high enough to cause stalemates. Thus, compared with the inefficiently low admission standards in the harmonious equilibrium, politicking by incumbent members can result in more efficient admission standards and thus greater long-term welfare.

Real-world organizations that fit our stylized model include academic departments, social clubs, professional societies, condominium associations, and partnership firms, etc. Our paper has two interesting implications about organizational design for such organizations. First, we provide a new rationale for the optimality of unanimity voting rule and for consensus-based decision-making requirements. Although unanimity voting may involve long decision processes, these processes result in a relatively balanced power structure and reasonably high admission standards for candidates of both types, which are good for the long-term welfare of the organization. Second, our finding suggests that there is an optimal degree of organizational incongruity. In a homogeneous organization, it is easy to make decisions but people tend to shirk in searching for high quality candidates. However, in a highly divided organization, internal politicking is so intense that decision-making processes are long and costly. A good organizational design should avoid these two extremes by trying to achieve the right degree of incongruity.

The rest of the paper is organized as follows. The next section reviews the literature. Section II presents the model and the solution concept, and Section III solves for two benchmarks: the first best solution of the model and the harmonious equilibrium in a politics-free club. In Section IV, we solve for the symmetric Markov equilibria
under both majority and unanimity voting rules. Section V derives the optimal voting rule and other implications for organizational design, and Section VI contains discussions and concluding remarks.

## I. Related Literature

To model dynamic interactions between internal politics and admission of new members, we draw on both collective search literature and dynamic club formation literature.

The dynamic club formation literature stems from the seminal work of Roberts (2015), who studies a dynamic model of club formation in which current members of the club vote by majority rule on whether to admit new members from a fixed population of potential members. ${ }^{2}$ Roberts defines Markov Voting Equilibrium (MVE) in this setting, and develops techniques to show the uniqueness of MVE and analyze the steady state of MVE. The theoretical analysis has been widely applied to investigate how distribution of political power evolves over time in contexts such as immigration law, suffrage, and constitutional rules (e.g., Jehiel and Scotchmer 2001; Lizzeri and Persico 2004; Jack and Lagunoff 2006; and Acemoglu, Egorov, and Sonin 2010, 2012, 2015). Our paper differs from this literature in several aspects. First, the existing literature focuses on how the club size is determined and who will be included in the club or excluded from it, while we study a dynamic club formation problem with a fixed club size. We are interested in the long-term power structure of the club, not the identities of the club members. Second, the existing literature is mostly abstract regarding what the club actually does besides voting on membership changes, while we embed a collective search problem in the dynamic club formation process to study how internal politics affects the search incentives of club members. Third, the existing literature emphasizes positive analysis of the evolution of club formation, while we develop a model with more payoff structures to allow for normative analysis of the optimal voting rule and of other issues of organizational design. ${ }^{3}$

In the collective search literature (see, e.g., Albrecht, Anderson, and Vroman 2010; Compte and Jehiel 2010; Moldovanu and Shi 2013), researchers consider a search problem where a once-and-for-all decision to stop searching is made by a voting committee that sequentially examines each available option. Committee members have different fixed preferences over the options, and collective decisions are made according to a prespecified voting rule. A main focus of this literature is to compare collective search with single-agent search, and to examine how committee composition and decision rules affect search outcomes. Differing from this literature, our paper studies an infinitely repeated problem of collective search in which the formation of the voting committee endogenously changes over time. In our model, the value of admitting a candidate to an incumbent member is endogenous, in the

[^1]sense that the change in power structure brought about by admitting him affects how members make future admission decisions and hence affects future payoffs.

In the literature, Schmeiser (2012) is most closely related to our paper. Schmeiser also considers a model in which existing board members vote to admit a new member to replace a randomly retired member. But unlike our model, in each period, two candidates are simultaneously observed: one insider and one outsider, and the organization must hire one of them. Therefore, there is no collective search, which is essential to our analysis and drives the intertemporal free-riding problem.

## II. The Game

## A. Model Setup

We consider an infinite-horizon game in discrete time indexed by $t=0,1,2, \ldots$ There is a club of fixed size $N=3 \cdot{ }^{4}$ In each period $t$, one of the incumbent members is chosen randomly to exit the club, and before this exit occurs, the three members must select one new member from a large pool of outside candidates who want to join the club. All of the players are risk-neutral and maximize their expected utility. For simplicity, we assume there is no discounting. ${ }^{5}$ We also normalize the outside option for each player to zero.

A player, either an incumbent member or a candidate, is characterized by his quality and his type. A player's quality, denoted by $v$, represents the skills, prestige, or resources that he can bring to the club and is valuable to the whole club. We suppose that a player of quality $v$ brings a common value of $v$ per period to every member of the club including himself, so his total contribution to the club value per period is $3 v$. Players in the population differ in quality. For the population, suppose $v$ is distributed according to a uniform distribution function $F(v)$ on $[\underline{v}, \bar{v}]$, where $0 \leq \underline{v}<\bar{v}$. When, in a given period, the club's members have qualities $v_{k}, k \in\{1,2,3\}$, each member's benefit from club membership in that period is $V=\sum_{k=1}^{3} v_{k}$.

Aside from quality heterogeneity, players belong to one of two types, "left" type and "right" type, and each type is equally represented in the population. Players' types are exogenously given and cannot be changed afterwards. ${ }^{6}$ Type is important because club politics are centered on such characteristics. We consider a situation with distributive politics in the following sense. In each period, there is a fixed amount of total rent $B$ in the club to be distributed to its members. We suppose for simplicity that members of the majority type share the rent equally among themselves. ${ }^{7}$

[^2]The total benefit to a club member in a period is the sum of his benefit from club membership $V$ and the rent he receives in that period. Formally, let a single variable $I \in\{0,1,2,3\}$ indicate the number of right types among the club's incumbent members. We will call $I$ the "state" of the club. The states can be further divided into two groups. Contentious states 1 and 2 are respectively called left-majority and right-majority states, and states 0 and 3 are respectively called left-homogeneous and right-homogeneous states. In a club with qualities $v_{k}, k \in\{1,2,3\}$, a right-type incumbent member's current period total benefit is $\sum_{i=1}^{3} v_{k=1}+(B / 3)$ in state 3 , is $\sum_{k=1}^{3} v_{k}+(B / 2)$ in state 2 , and is $\sum_{k=1}^{3} v_{k}$ otherwise. ${ }^{8}$

Each period $t$ is divided into three stages. The first (selection) stage may consist of an infinite number of rounds. In each round, a candidate is randomly drawn from the population. His quality and type are then revealed to the incumbent members, who then vote whether to accept him as a new member. Under majority (unanimity) rule, if a candidate gets two (three) or more yes votes, then he is admitted to the club and the selection stage of the current period is over. ${ }^{9}$ If a candidate does not get the required yes votes, then the club draws another candidate from the population and goes through the same procedure. This selection process continues until a candidate is admitted. We suppose that each selection round imposes a cost of $\tau>0$ to every incumbent member. ${ }^{10}$ Since member selection takes at least one round, we count selection costs only if it takes more than one round. A given period can never end if the selection stage continues forever. Hence, we include negative infinity $(-\infty)$ in the range of each member's payoffs.

After the admission of a new member, in the second (exiting) stage, one of the incumbent members is chosen randomly to exit the club permanently for exogenous reasons (e.g., natural death, family reasons). In the third (political decision) stage, the two remaining incumbent members and the new member together decide on club politics (e.g., the distribution of rent $B$ ). And then, each member in the club receives his current period total benefit (including the benefit from club membership and the rent). The same process repeats in each period infinitely.

The recruitment of new academic faculty members provides an excellent context for our model. On the one hand, academic departments are viewed as "status organizations" in the sense of Hansmann (1986) and Prüfer and Walz (2013): each member's quality is valuable to every member of the organization. On the other hand, the recruitment decision often involves lengthy deliberations due to disagreements. The recruitment of Milton Friedman by the Chicago Economics Department in 1946 is a concrete example. According to Mitch (2016), this decision involved "a fairly intense struggle between Knight and his former students on the one side and the Cowles Commission and its adherents on the other." The struggle was caused by

[^3]differences in research methodology and political ideology, which can be viewed as corresponding to the predetermined types in our model.

We want to make several remarks about our model setting. First, the sequence of move within each period as specified above is convenient for our analysis because it ensures that there are an odd number of voting members in both the candidate selection stage and the club's political decision stage. One can think of alternative sequences of move. For example, suppose that at the beginning of each period the club has four members, and one of the incumbent members exits. The three remaining members vote to admit a new member. Then in the political decision stage, the four members vote under majority rule with a prespecified tie-breaking rule. With some minor modifications to the solution of our model, our qualitative results should still hold with this alternative sequence of move.

Second, in the above formulation of club politics, we make two assumptions. One is about the nature of "incomplete contracts," namely, there are certain rents of the club that cannot be specified clear enough in contracts among club members and hence are subject to ex post negotiations by the members. This should be true for most organizations, otherwise there is little point in setting up an organization if all of its resources and rents are completely predetermined in contracts. Moreover, as our results will show, it is actually not always in the best interest of the club to predetermine rent distribution even if all rents are contractible.

Another assumption in our formulation of club politics is that, by distributive politics, the total amount of rent is constant in each period independent of power structure. This assumption is likely to be satisfied in applications where the discretionary resources of the organization are more or less fixed, e.g., research funds, office spaces, or prestigious positions. By this assumption, the total value of the club depends only on the quality of its members and is independent of its power structure, which greatly facilitates welfare comparison. One implication of fixed total rent is that each member of the majority type gets a smaller share of the total rent as the majority increases. Thus, majority-type incumbents would favor candidates of the opposite type if they are assured of keeping control over the internal politics of the club. In the online Appendix, we show that our main results are very robust to other specifications of rent allocation.

Finally, we suppose that the club's voting rule for admitting new members is fixed at the beginning of the game and cannot be modified later. This is of course for analytical simplicity, but it is also consistent with the observation that many organizations have very strict requirements for changing their chapter rules or constitutions. Our central interest is in finding the best voting rule for admitting new members in terms of the long-term welfare of the club.

## B. Strategies and Solution Concepts

Throughout the paper, we focus on symmetric Markov equilibria with weakly stage-undominated strategies of the game. Without putting restrictions on strategies, the game admits trivial equilibria in the following sense. In any given period, if every incumbent member votes "no" on any candidate, then it is indeed an equilibrium in which no candidate will be admitted forever. But in this equilibrium, every incumbent
member gets a payoff of negative infinity. Using this equilibrium as a punishment, then any outcome can be supported in equilibrium. By focusing on Markovian strategies, we rule out such trivial equilibria by ruling out history-dependent award and punishment schemes.

In any period, each incumbent member's strategic decision is to vote on whether to accept a candidate or not in the selection stage. There are no actions to be taken in the exiting stage or the political decision stage. In general, following Maskin and Tirole (2001), an incumbent member's Markovian voting decision in a selection stage can depend on all of the payoff-relevant variables including the quality and type of the candidate, and the quality and type profiles of the three incumbent members. More formally, a Markovian strategy of an incumbent $k=1,2,3$ can be written as

$$
\sigma_{k}:[\underline{v}, \bar{v}]^{3} \times[\underline{v}, \bar{v}] \times\{L, R\}^{3} \times\{l, r\} \rightarrow\{y e s, n o\},
$$

where the four determinants of the mapping are the quality profile of the incumbents, the quality of the candidate, the type profile of the incumbents, and the type of the candidate, respectively. Denote $\sigma=\left\{\sigma_{k}\right\}_{k=1}^{\}}$to be the combination of the three incumbent members' strategies.

Let $b \in\{L, R\}$ denote the type of an incumbent member, and $b^{\prime} \in\{l, r\}$ denote the type of a candidate. We use different notations for the set of types of current members and candidates simply to avoid possible confusion when notating the value functions. Each strategy $\sigma_{k}$ determines an incumbent $k$ 's acceptance region $\mathcal{A}_{k} \subset[\underline{v}, \bar{v}] \times\{l, r\}$. Given the strategy profile $\sigma=\left\{\sigma_{k}\right\}_{k=1}^{\}}$and the club's voting rule, we can uniquely determine the club's acceptance region $\mathcal{A}_{k} \subset[\underline{v}, \bar{v}] \times\{l, r\}$. This in turn determines the expected quality of the newly admitted candidate $E\left[v_{\text {new }} \mid \sigma\right]$, each member's expected rent conditional on his survival $E\left[\mu \mid\left\{b_{k}\right\}, \sigma\right]$, and the expected search length (or expected delay) $E[d \mid \sigma]$. Notice that the expected rent also depends on the incumbent members' type profile $\left\{b_{k}\right\}$.

At any time $t_{0}$, suppose that the quality profile of the incumbent members is $\left\{v_{k}\right\}_{k=1}^{3}$ and their type profile is $\left\{b_{k}\right\}_{k=1}^{3}$. For a given admission strategy profile $\sigma$, we can calculate the total expected payoff of an incumbent member $k$. Denote this value as $u_{k}\left(\left\{v_{k}\right\}_{k=1}^{3},\left\{b_{k}\right\}_{k=1}^{3}, \sigma\right)$, and $u_{k}$ can be determined recursively as
(1) $u_{k}\left(\left\{v_{k}\right\}_{k=1}^{3},\left\{b_{k}\right\}_{k=1}^{3}, \sigma\right)=\frac{2}{3}\left\{v_{k}+E\left[\mu \mid\left\{b_{k}\right\}, \sigma\right]+E\left[v_{\text {new }} \mid \sigma\right]\right.$

$$
\begin{aligned}
& \left.+\frac{1}{2} \sum_{j \neq k}\left[v_{j}+E\left[u_{k}\left(\left\{v_{k}, v_{j}, v_{\text {new }}\right\},\left\{b_{k}, b_{j}, b_{\text {new }}\right\}, \sigma\right) \mid \sigma\right]\right]\right\} \\
& \quad-\tau E[d \mid \sigma] .
\end{aligned}
$$

The interpretation is as follows. The term $2 / 3$ is the probability that member $k$ survives one period, otherwise member $k$ exits the club and gets the normalized outside option of zero. In the bracket, $v_{k}$ is member $k$ 's own quality, $E\left[\mu \mid\left\{b_{k}\right\}, \sigma\right]$ represents
member $k$ 's expected rent in this period, and $E\left[v_{\text {new }} \mid \sigma\right]$ is the expected quality of the newly admitted member. Conditional on member $k$ 's survival, $1 / 2$ is the probability that any other member $j \neq k$ survives one period. If member $j$ survives, member $k$ receives $v_{j}$, member $j$ 's quality, and the expected value in the next period, which is $E\left[u_{k}\left(\left\{v_{k}, v_{j}, v_{\text {new }}\right\},\left\{b_{k}, b_{j}, b_{\text {new }}\right\}, \sigma\right) \mid \sigma\right]$. The last term $\tau E[d \mid \sigma]$ is the expected search cost to member $k$ in this period. We allow the possibility that $E[d \mid \sigma]=\infty$ such that $u_{k}=-\infty$.

Equation (1) provides a recursive definition of value function $u_{k}$. To solve the model, we compute the value functions by deduction. For a given admission strategy profile $\sigma$, we can calculate the expected payoff of an incumbent member, say $k=1$, who is of type $b \in\{L, R\}$, at time $t=t_{0}$ as

$$
E u_{1}\left(t=t_{0}\right)=\frac{2}{3}\left[v_{1}+\frac{1}{2} \sum_{k=2}^{3} v_{k}+E\left[v_{n e w}^{t_{0}} \mid \sigma\right]+E\left[\mu^{t_{0}} \mid\left\{b_{k}^{t_{0}}\right\}, \sigma\right]\right]-\tau E\left[d^{t_{0}} \mid \sigma\right]
$$

Similarly, member 1's expected payoff in the next period $t=t_{0}+1$, $E u_{1}\left(t=t_{0}+1\right)$, is given by

$$
\begin{aligned}
\left(\frac{2}{3}\right)^{2}[ & v_{1} \\
& \left.+\frac{1}{2}\left[\frac{1}{2} \sum_{k=2}^{3} v_{k}+E\left[v_{\text {new }}^{t_{0}} \mid \sigma\right]\right]+E\left[v_{\text {new }}^{t_{0}+1} \mid \sigma\right]+E\left[\mu^{t_{0}+1} \mid\left\{b_{k}^{t_{0}+1}\right\}, \sigma\right]\right] \\
& -\tau E\left[d^{t_{0}+1} \mid \sigma\right] .
\end{aligned}
$$

By deduction, member 1's value function $u_{1}$ under strategy profile $\sigma$ can be written as

$$
\begin{align*}
u_{1}\left(\left\{v_{k}\right\}_{k=1}^{3},\left\{b_{k}\right\}_{k=1}^{3}, \sigma\right) & =\sum_{t=t_{0}}^{\infty} E u_{1}(t)  \tag{2}\\
& =\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^{n} v_{1}+\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^{n}\left(v_{2}+v_{3}\right)+\pi_{1}\left(\left\{b_{k}\right\}, \sigma\right) \\
& =2 v_{1}+\frac{1}{2}\left(v_{2}+v_{3}\right)+\pi_{1}\left(\left\{b_{k}\right\}, \sigma\right)
\end{align*}
$$

where the search payoff $\pi_{1}\left(\left\{b_{k}\right\}, \sigma\right)$ contains all the terms related to the expected qualities of newly admitted members in each period, the expected rent member 1 gets in each period, and the expected search cost in each period. In other words, $\pi_{1}\left(\left\{b_{k}\right\}, \sigma\right)$ is the expected value incumbent member 1 can obtain through the club's admission of new members in current and future periods. It is also worth noting that the coefficient for $v_{1}$ and those for $v_{2}$ and $v_{3}$ are different. This is because member 1 has to stay in the club to enjoy positive benefits. But conditional on member 1's survival, the survival probability for the other two has to be lower.

A key observation is that the quality profile $\left\{v_{k}\right\}$ enters each incumbent member's value function as a constant, and does not directly affect $\pi_{k}\left(\left\{b_{k}\right\}, \sigma\right)$. We can hence
simplify member $k$ 's Markovian strategies to be independent of the incumbent members' quality profile

$$
\sigma_{k}:[\underline{v}, \bar{v}] \times\{L, R\}^{3} \times\{l, r\} \rightarrow\{\text { yes }, n o\} .
$$

We focus on symmetric Markov equilibria with weakly stage-undominated strategies of the game. Since strategies do not depend on the quality profile, members of the same type will choose the same admission strategy in a symmetric equilibrium. Moreover, the distribution of rent is purely determined by the state variable $I$, defined as the number of right-type incumbent members. Therefore, an incumbent member of the same type $b$ will receive the same search payoff in state $i$, and we denote this payoff $\pi_{i}^{b}(\sigma)$. The strategies of member $k$ who is type $b$ in state $i$ can be rewritten as

$$
\sigma_{i}^{b}:[\underline{v}, \bar{v}] \times\{l, r\} \rightarrow\{\text { yes }, n o\} .
$$

Moreover, we say that incumbent member $k$ with type $b$ in state $i$ votes sincerely if this member accepts any candidate with characteristics $\left(\tilde{v}, b^{\prime}\right)$ if and only if

$$
\begin{equation*}
\pi_{i}^{b}(\sigma)-\tau \leq \frac{2}{3}\left[\tilde{v}+E\left[\mu \mid b, i, b^{\prime}\right]+E\left[\pi_{i^{\prime}}^{b} \mid i, b^{\prime}\right]\right] \tag{3}
\end{equation*}
$$

where $E\left[\mu \mid b, i, b^{\prime}\right]$ represents member $k$ 's expected rent from admitting a type $b^{\prime}$ candidate in state $i$ and $i^{\prime}$ is the state in the next period. The right-hand side expression of condition (3) is the total expected search payoff to member $k$ from admitting the candidate with characteristics $\left(\tilde{v}, b^{\prime}\right)$ right away, and the left-hand side expression is his total expected search payoff from rejecting the candidate and searching for another candidate in the next round. ${ }^{11}$ So the condition requires sincere voting in the selection stage. This is needed to rule out trivial voting equilibria. ${ }^{12}$ The sincere voting condition implies that an equilibrium admission strategy profile $\sigma_{i}^{b}$ should take the following cutoff form (minimal admission standards): a right incumbent in state $i$ votes "yes" on a candidate of types $b^{\prime} \in\{l, r\}$ if the candidate's quality is higher than a quality standard $v_{i}^{b^{\prime}}$ such that ${ }^{13}$

$$
v_{i}^{b^{\prime}} \begin{cases}=\underline{v} & \text { if } \pi_{i}^{R}\left(v_{i}^{b^{\prime}}=\underline{v}, y e s\right) \geq \pi_{i}^{R}-\tau  \tag{4}\\ \in(\underline{v}, \bar{v}) & \text { if } \pi_{i}^{R}\left(v_{i}^{b^{\prime}}, \text { yes }\right)=\pi_{i}^{R}-\tau\end{cases}
$$

[^4]where $\pi_{i}^{R}\left(v_{i}^{b^{\prime}}\right.$, yes $)$ is the expected search payoff from admitting $v_{i}^{b^{\prime}}$ :
$$
\frac{2}{3}\left[v_{i}^{b^{\prime}}+E\left[\mu \mid R, i, b^{\prime}\right]+E\left[\pi_{i^{\prime}}^{R} \mid i, b^{\prime}\right]\right] .
$$

Finally, we can formally define our solution concept.
DEFINITION 1: A symmetric Markov equilibrium with weakly stage-undominated strategies (henceforth, referred to as an "equilibrium") consists of a combination of strategy and search payoff functions $\left(\sigma_{i}^{b}, \pi_{i}^{b}\right)$ that satisfy the following conditions:
(i) For each state $i$ and type $b, \pi_{i}^{b}$ satisfies the following equation:

$$
\begin{equation*}
\pi_{i}^{b}=\frac{2}{3}\left[E\left[v_{\text {new }} \mid \sigma\right]+E[\mu \mid b, i, \sigma]+E\left[\pi_{i^{\prime}}^{b} \mid i, \sigma\right]\right]-\tau E[d \mid \sigma] \tag{5}
\end{equation*}
$$

where $E[\mu \mid b, i, \sigma]$ represents type- $b$ member's expected rent in state $i$ and $i^{\prime}$ is the state in the next period;
(ii) Denote $\tilde{b}$ to be the opposite type of type $b$, and then $\sigma_{i}^{b}\left(v, b^{\prime}\right)=\sigma_{3-i}^{\tilde{b}}\left(v, \tilde{b}^{\prime}\right)$;
(iii) $\sigma_{i}^{b}$ takes the cutoff form represented by equation (4).

Since the model is symmetric with respect to the two types, right-type incumbents in state $i$ are in the same strategic position as left-type incumbents in state $3-i$. Condition (ii) in the above definition requires that in equilibrium, when facing a type $b^{\prime}$ candidate, right-type incumbents in state $i$ choose the same strategies as left-type members in state $3-i$, facing the opposite type candidate. By this condition, we only need to specify a right-type incumbent's equilibrium cutoffs $\left(v_{i}^{r}, v_{i}^{l}\right)$, where $r, l$ is the candidate's type, because a left-type incumbent's equilibrium cutoffs in state $i$ are those of the right-type incumbent in state $3-i$. Together with the voting rule, an equilibrium admission strategy profile, now consisting of the quality standards for the two types of candidates set by the two types of incumbents, determines the club's equilibrium admission policy. Under majority voting, the equilibrium admission policy is always the same as the majority-type incumbent's equilibrium cutoffs; while under unanimity voting, the equilibrium admission policy in a contentious state is the larger of the equilibrium cutoffs set by the two types of incumbents.

Given the complexity of the model, even with so many restrictions on equilibrium strategies, there may still be multiple equilibria. We solve this problem by selecting the equilibrium with the greatest long-term welfare, which is defined in the online Appendix. ${ }^{14}$

[^5]
## III. Two Benchmarks

## A. The First Best Solution

In this section, we solve for the first best solution for the club as a benchmark case. Since the amount of rent is fixed, the founder or social planner of the club should maximize the club's per period welfare, which is simply the sum of the quality of the club members minus the total search costs, and thus is independent of its power structure. It is easy to see that the social planner's optimal admission policy should be the same for the two symmetric types, and take the following cutoff form: admit a candidate if and only if his quality is at least $v^{*}$. Since every member of the club is admitted by such a policy, each member's expected benefit from club membership per period is $3 E\left[v \mid v \geq v^{*}\right]$. To calculate the expected search cost in each period, note that the probability that a candidate is admitted is $x^{*}=1-F\left(v^{*}\right)$. Given $v$ is uniformly distributed on $[\underline{v}, \bar{v}]$, denote $a \equiv \bar{v}-\underline{v}$ to be the spread of the quality distribution and $F\left(v^{*}\right)=\left(v^{*}-\underline{v}\right) / a$. Hence, the expected delay in each period is

$$
E\left[d^{*}\right]=\sum_{d=1}^{\infty} x^{*}\left(1-x^{*}\right)^{d} d=\left(1-x^{*}\right) / x^{*}=F\left(v^{*}\right) /\left(1-F\left(v^{*}\right)\right)
$$

Each member's expected net value per period is therefore $3 E\left[v \mid v \geq v^{*}\right]-$ $\tau F\left(v^{*}\right) /\left(1-F\left(v^{*}\right)\right)$. Maximizing this function, we obtain the following proposition (proof omitted).

PROPOSITION 1: In the first best solution,
(i) when $\tau \geq 3 a / 2$, the club admits any candidate (i.e., $v^{*}=\underline{v}$ ).
(ii) when $\tau<3 a / 2$, the club admits candidates whose quality is above $v^{*}$, where $v^{*}=\bar{v}-\sqrt{2 a \tau / 3}$.

In the optimal policy, the social planner trades off the cost of delay and the benefits of setting a high standard to get high quality candidates. When search is very costly, the club admits any candidate to avoid paying the search cost. When the unit search $\operatorname{cost} \tau$ is not too high, the social planner has an optimal interior searching rule: she will search until she finds a candidate whose quality is above a prefixed level $v^{*}$.

In the interior solution $\left(v^{*}=\bar{v}-\sqrt{2 a \tau / 3}\right)$, the probability that a candidate is admitted in the first best solution can be expressed as $x^{*}=\left(\bar{v}-v^{*}\right) / a$ $=\sqrt{2 \tau /(3 a)}$. This has a very simple interpretation. The smaller $a$ is, the smaller is the benefit of searching for one more round. ${ }^{15}$ Thus, the admission probability will be higher (or the admission standard will be lower) if the unit search $\operatorname{cost} \tau$ is higher or the quality distribution has a smaller spread. The club's expected net value in the

[^6]first best solution can be calculated as $U^{*}=3 E v+1.5 a-\sqrt{6 a \tau}+\tau$, where $E v$ denotes the expectation of $v$.

## B. Equilibrium without Internal Politics

We now consider another benchmark case in which internal politics are of no importance. This happens when $B=0$ or equivalently, when the club's rent is predetermined and not subject to the internal politicking of its members. In such a case, all incumbent members have identical preferences over admission policies since they care only about the candidate's quality. In an equilibrium with weakly stage-undominated strategies, they only need to solve for the optimal admission policy that maximizes their payoffs. We call the equilibrium in this case the "harmonious equilibrium." In the harmonious equilibrium, the incumbent members need to solve an optimal stopping problem: admit a candidate if and only if his quality is at least $\hat{v}$.

The expected value to an incumbent member if a candidate with quality $\hat{v}$ is admitted is ${ }^{16}$

$$
\frac{2}{3}\left(1+\frac{1}{3}+\left(\frac{1}{3}\right)^{2}+\cdots\right) \hat{v}=\hat{v}
$$

Let $w$ be the expected net value an incumbent member can obtain from selecting a new member using the optimal rule. Clearly, $w \in[\underline{v}, \bar{v}] \cdot{ }^{17}$ By the definition of $\hat{v}$, it must be that

$$
\begin{equation*}
\hat{v}=\max \{w-\tau, \underline{v}\} \tag{6}
\end{equation*}
$$

When $w-\tau \geq \underline{v}$, this means that if the candidate's quality happens to be $\hat{v}$, the incumbent members must be indifferent between admitting him now (i.e., receiving value $\hat{v}$ ) and rejecting and waiting to see another candidate in the next round. In the latter case, an incumbent will receive a value of $w$ (from the same optimal admission policy next round) but will incur the waiting cost of $\tau$. When $w-\tau<\underline{v}$, waiting never makes sense so the club should admit any candidate, that is, set $\hat{v}=\underline{v}$.

By the definition of $w$, we have

$$
\begin{equation*}
w=\int_{\hat{v}}^{\bar{v}} v d F(v)+F(\hat{v})(w-\tau) \tag{7}
\end{equation*}
$$

where the first term is the expected value in the event that the candidate's quality is above $\hat{v}$ (so he is admitted), and the second term is the expected net value in the event that the candidate's quality is below $\hat{v}$ (so the club has to search further).

[^7]Equations (6) and (7) define the optimal $\hat{v}$ and the resulting expected net value $w$. We have the following result (proof omitted).

PROPOSITION 2: The club's optimal admission policy in the harmonious equilibrium can be characterized as follows:
(i) when $\tau \geq a / 2$, the club admits any candidate (i.e., $\hat{v}=\underline{v})$.
(ii) when $\tau<a / 2$, the club admits candidates whose quality is above $\hat{v}$, where $\hat{v}=\bar{v}-\sqrt{2 a \tau}$.
(iii) when $\tau<3 a / 2$, the admission standard in the harmonious equilibrium is strictly lower than the first best standard.

The characterization of the harmonious equilibrium in Proposition 2 is easy to understand. What is interesting is that even in a politics-free club, the club's admission policy is inefficient. In the harmonious equilibrium, an incumbent member only gets a marginal benefit of $v$ from admitting a candidate with quality $v$, while the social planner's marginal benefit from admitting this candidate is $3 v$. Facing the same marginal search cost, an incumbent member in the harmonious equilibrium thus sets a lower standard than the social planner. This is similar to the under-provision of public goods in the standard static model of clubs. However, in our model, inefficiency does not come from free riding among incumbent members in a given period. The joint surplus of all incumbent members in any given period is maximized in the harmonious equilibrium. The source of inefficiency in the harmonious equilibrium is intertemporal free-riding, because incumbent members in the current period do not take into account the benefits of having high quality new members to future generations of club members. Thus, they search less relative to the efficient level by having lower admission standards. ${ }^{18}$

In the interior solution $(\hat{v}=\bar{v}-\sqrt{2 a \tau})$, the probability that a candidate is admitted in the harmonious equilibrium is $\hat{x}=(\bar{v}-\hat{v}) / a=\sqrt{2 \tau / a}$. The club's expected net value in the harmonious equilibrium is $U^{h}=3 E v+1.5 a-$ $2 \sqrt{2 a \tau}+\tau$, which is strictly lower than that in the first best solution.

## IV. Equilibria with Internal Politics

In this section, we characterize the most efficient equilibrium of the model with internal politics under majority voting and unanimity voting. To avoid trivial corner solutions, we suppose that the unit search cost $\tau$ is less than $a / 4$. This assumption appears to be reasonable in most applications, because the selection costs involved in recruiting one candidate, such as the time costs of reading files and going to meetings, should be small relative to the importance of admitting high quality new members.

[^8]
## A. Equilibrium under Majority Voting

Under majority voting, the majority type in the current period determines the equilibrium admission policy in that period. Specifically, in each of the four states (left-homogeneous, right-homogeneous, left-majority, right-majority), the majority type decides the minimal quality standards necessary to admit left- and right-type candidates. In a symmetric equilibrium, four admission standards need to be determined: admission standards for left- and right-type candidates in the right-homogeneous state and the right-majority state, and then admission standards in the left-homogeneous state and the left-majority state can be found identically for the opposite types of candidates. Fixing admission policies, the value functions of type- $b$ incumbent members, $\pi^{b}$, can be calculated in the way described in the previous section. With these value functions, we can analyze the optimal admission policy for the incumbent members in each state. The detailed steps for characterizing the most efficient equilibrium and the proofs of our results in the remainder of the paper are relegated to the Appendix.

Intuitively, when deciding whether to admit a candidate, the majority-type incumbents take into account four factors: (i) his qualifications, (ii) the search cost (if he is to be rejected and another candidate sought), (iii) the effect on rent allocation in the current period, and (iv) the effect on the future power structure of the club. Factor (iv) depends on how the power structure evolves in the future, which depends on admission policies in different states. For example, in a right-majority state, the right-type incumbents may lose control of the rent distribution in the current period if admitting a left-type candidate, as well as future control of the club (if the left-type incumbents, when in power, do not admit right-type candidates).

In equilibrium, it turns out that the admission policies and the pattern of power changes hinge on one simple variable, which is defined as $c \equiv B /(12 \sqrt{a \tau})$. The variable $c$ can be interpreted as the club's degree of incongruity. It is small (or, the club is congruous) when the rent $B$ (the gain from politicking) is small, or when admitting high quality candidates is important (the uncertainty of candidate quality $a$ is relatively large), or when delay is costly ( $\tau$ is relatively large).

The next proposition characterizes the most efficient equilibrium under majority voting.

## PROPOSITION 3: Under majority voting rule,

(i) when the club is relatively congruous $(0<c<0.43)$, the most efficient equilibrium is the "power-switching equilibrium" in which both types of candidates are admitted with positive probabilities so that power moves back and forth between the two types;
(ii) when the club is relatively incongruous $(c>0.43)$, the most efficient equilibrium is the "glass-ceiling equilibrium" in which, in contentious states, the majority-type incumbents will never admit candidates of the opposite type, and thus the club will never experience a change of power.

Proposition 3 says that the pattern of the most efficient symmetric equilibrium under majority voting crucially depends on the degree of incongruity $c$. Intuitively, when the club is relatively congruous ( $c<0.43$ ), searching for better candidates is more important than grabbing rent through internal politics, so there exists a power-switching equilibrium where majority-type incumbents will admit candidates of the opposite type who are of high quality. However, if the club is relatively incongruous ( $c>0.43$ ), controlling rent allocation becomes the dominant concern for the majority-type incumbents in contentious states, so they do not admit candidates of the opposite type no matter how qualified they are. Consequently, in such a case, the only equilibrium is the glass-ceiling equilibrium, in which the type that controls rent allocation at the very beginning will always hold power in the club, and the minority type will never have a real say in the internal politics. ${ }^{19}$ In the region when $c \in(10 / 29,0.43)$, both power-switching and glass-ceiling equilibria exist, and the welfare comparison favors the power-switching equilibrium. In the power-switching equilibrium, the majority-type incumbents in contentious states are still willing to admit high quality minority-type candidates, because of their high quality and also because of the need to reduce undue search costs. For relatively small $c$, this equilibrium leads to less distortion in admission policies and greater long-term welfare than does the glass-ceiling equilibrium.

Internal politics distort admission policies. To be more explicit about the distortions, we compare admission probabilities instead of admission standards. Recall that the admission probability for any type of candidate is $x^{*}=\sqrt{2 \tau /(3 a)}$ in the first best and is $\hat{x}=\sqrt{2 \tau / a}$ in the harmonious equilibrium in the absence of internal politics. Define $x_{i}^{b^{\prime}} \equiv\left(\bar{v}-v_{i}^{b^{\prime}}\right) / a$ as the probability that a type $b^{\prime}$ candidate will be admitted in state $i$ in an equilibrium under internal politics, where $v_{i}^{\prime}$ is the admission standard. The proof of Proposition 3 also implies the following corollary.

COROLLARY 1: In both the power-switching and glass-ceiling equilibria, the majority-type incumbents in contentious states favor candidates of their own type and discriminate against candidates of the opposite type: $x_{2}^{r}>\hat{x}=\sqrt{2 \tau / a}>x_{2}^{l}$; but in homogeneous states, they have lower standards for the opposite type than for their own type: $x_{3}^{l}>\hat{x}=\sqrt{2 \tau / a}>x_{3}^{r}$. Moreover, the distortions are greater in contentious states than in homogeneous states: $x_{3}^{l}-x_{3}^{r} \leq x_{2}^{r}-x_{2}^{l}$.

Corollary 1 says that under majority rule, the candidates of the majority type always have a higher probability of being admitted in contentious states but a lower probability in homogeneous states. In contentious states, the majority-type incumbents fear that admitting a candidate of the opposite type may shift the balance of power against them and hence set much higher standards for candidates of the opposite type than for those of their own type. When the club is relatively incongruous, the majority-type incumbents' discrimination goes to the extreme and candidates of

[^9]the opposite type are completely excluded. In contrast, when all three members are of the right (or left) type, they are safely in control of the power over rent distribution. Since they prefer sharing rent with fewer members of their own type, they will favor candidates of the opposite type and discriminate against those of their own type. The distortion of admission standards is smaller in homogeneous states than in contentious states because majority-type incumbents do not need to worry about losing control over rent allocation in the current period.

## B. Equilibrium under Unanimity Voting

Under unanimity voting rule, all incumbent members need to reach a consensus about admitting a candidate. This is easily achieved in homogeneous states. But in contentious states, the majority- and minority-type incumbents will have different standards for each type of candidate, and the admission criterion is set by the higher of the two standards held by the the two types of incumbent members.

Using an approach similar to that used in solving for equilibria under majority rule, we can characterize the most efficient equilibrium under unanimity voting rule.

PROPOSITION 4: Under unanimity voting rule,
(i) when the club is congruous $(0<c<0.47)$, the most efficient equilibrium is the "pro-minority power-switching" equilibrium in which candidates of both types are admitted in each state with positive probabilities, but candidates of the majority type in contentious states have a lower probability of being admitted than those of the minority type;
(ii) when the degree of incongruity is intermediate $(0.47<c<1.97)$, the most efficient equilibrium is the "pro-majority power-switching" equilibrium in which candidates of the majority type in contentious states have a higher probability of being admitted than those of the minority type;
(iii) when the club is very incongruous ( $c>1.97$ ), the most efficient equilibrium is the glass-ceiling equilibrium.

Similar to Proposition 3, the most efficient equilibrium under unanimity rule also involves power-switching when the club is relatively congruous and glass-ceiling when the club is relatively incongruous. However, there are important differences between the two cases.

Figure 1 below depicts the normalized admission probabilities in the rightmajority state when the degree of incongruity $c$ is small $(c<0.43)$ in the most efficient equilibrium. ${ }^{20}$ We choose the parameter region $c<0.43$ to guarantee that the most efficient equilibrium is the "power-switching equilibrium" under majority voting.

[^10]

Figure 1. Equilibrium Normalized Admission Probabilities
Note: Panel A shows equilibrium normalized admission probabilities $\left(\times \sqrt{\frac{a}{4 \tau}}\right)$ in state 2 (contentious state with majority dominance) under majority voting; panel B shows equilibrium normalized admission probabilities in state 2 under unanimity voting.

As shown in Figure 1, under majority rule, the "power-switching" equilibrium favors the majority-type candidates in the sense that candidates of the majority type are admitted with higher probability (lower standard) than those of the minority type in contentious states. However, under unanimity rule, candidates of the left type are more likely to be admitted than those of the right type in the right-majority state. We call this the "pro-minority power-switching" equilibrium to emphasize the contrast to the "pro-majority power-switching" equilibrium. In the pro-minority power-switching equilibrium, the minority incumbent member has more power than his majority peers in selecting new members. This is because under unanimity rule, the minority-type incumbent has strong incentives to block the majority-type candidates in contentious states, so that he may gain the control over the rent allocation. In contrast, each majority member still has a 50 percent chance of being in power after the admission of a minority-type candidate (conditional on his remaining in the club), and hence he has weaker incentives to block the minority-type candidates. Thus, when the club is congruous (admitting high-quality candidates is more important than controlling rent allocation), the majority-type incumbents will avoid fighting with the minority-type incumbent, and hence the minority member can take
advantage of this to admit his favored candidate with higher probability. It can be shown that the "pro-minority power-switching" equilibrium is the unique (hence trivially the most efficient) equilibrium when $c$ is small, as stated in Proposition 4(i).

Proposition 4(ii) gives the range of $c$ in which the pro-majority power-switching equilibrium is the most efficient, by combining the ranges of $c$ in which it dominates either the pro-minority power-switching equilibrium or the glass-ceiling equilibrium, or both. ${ }^{21}$ Both pro-majority and pro-minority power-switching equilibria exist when the degree of incongruity $c$ falls into an intermediate range $(10 / 29<c<2 / 3)$. This coexistence is like a Game of Chicken. In contentious states, the minority-type incumbent will be tough on majority candidates if he expects the majority-type incumbents to be soft on minority ones, which leads to the pro-minority power-switching equilibrium, and vice versa. When both exist, the efficiency comparison of the two equilibria depends on the degree of incongruity. For a small range of $c(10 / 29<c<0.47)$, both equilibria exist, but the pro-minority power-switching equilibrium dominates in efficiency. This case is included in Proposition 4(i). As $c$ increases, in the pro-minority power-switching equilibrium, the minority-type incumbent keeps raising the admission standard for the majority-type candidates in contentious states (see Figure 1), resulting in greater and greater welfare loss. On the contrary, in the pro-majority power-switching equilibrium, when $c$ increases, the distortion of admission standards by the majority-type incumbents does not increase as fast as the distortion by minority-type incumbents in the pro-minority power-switching equilibrium, because it is less likely that the majority-type incumbents will lose power than that the minority-type incumbent will gain power in contentious states. Therefore, for $c>0.47$, when both exist, the pro-majority power-switching equilibrium dominates the pro-minority power-switching equilibrium in efficiency.

As in the case of majority voting (Proposition 3), the welfare comparison between the pro-majority power-switching equilibrium and the glass-ceiling equilibrium favors the former for relatively small $c(c<1.97)$. The intuition is exactly the same as in the case of majority voting, but this region is much larger in the case of unanimity voting. Under majority rule, the pro-majority power-switching equilibrium ceases to exist when $c$ is larger than 0.43 , but under unanimity rule, this equilibrium exists for a much wider range of $c$.

The pro-majority power-switching equilibrium continues to exist even when $c$ is very large. But Proposition 4(iii) says that for sufficiently large $c$, the most efficient equilibrium is the glass-ceiling equilibrium, which can be viewed as an extreme of the pro-majority power-switching equilibrium. Intuitively, when internal politics are very important, in contentious states, the majority-type incumbents are reluctant to admit candidates of the opposite type because they expect that power will be difficult to regain once they lose it, and the minority-type incumbent wants to insist on high standards for candidates of the majority type because control over rent allocation is too important to give up. Thus, there will be large political costs in contentious states. And this will make incumbents in homogeneous states hesitant to admit

[^11]candidates of the opposite type. Therefore, as the club becomes very incongruous, candidates will face very stringent admission standards (except those of the same type as the incumbents in homogeneous states), and hence the club experiences inefficiently long delays in selecting new members. As a result, for sufficiently large $c$, the most efficient equilibrium switches to the glass-ceiling equilibrium, which mitigates internal politics by eliminating the chance of the minorities having a say.

## V. Optimal Voting Rule and Organizational Design

Using the equilibrium characterization results of the preceding section, in this section we investigate the optimal voting rule and other organizational design issues. Naturally, we suppose that the founder or social planner of the club adopts the voting rule that yields the greatest long-term welfare for the club.

Equation (A.5) in the online Appendix gives the definition of long-term welfare for the club. It can be shown that the welfare function in the cases we are interested in can be expressed as

$$
U=3 E v+\frac{3}{2} a+\tau-2 \sqrt{a \tau} \gamma
$$

where $\gamma$ summarizes the total long-term expected welfare of the club in each case. Aside from the model's parameters, the welfare function of the club depends only on $\gamma$ : the smaller $\gamma$ is, the more efficient it is for the club. This greatly simplifies our welfare caparison.

It can be calculated that in the first best solution $\gamma^{*}=\sqrt{6} / 2$, and in the harmonious equilibrium $\hat{\gamma}=\sqrt{2}>\gamma^{*}$. In the case of majority voting,

$$
\gamma^{m} \equiv 4 q_{3} /\left(y_{3}^{r}+y_{3}^{l}\right)+4 q_{2} /\left(y_{2}^{r}+y_{2}^{l}\right),
$$

where $q_{3}\left(q_{2}\right)$ is the long-term stationary probability of the club being in the righthomogeneous (majority) state, and $y_{i}^{b^{\prime}}=x_{i}^{b^{\prime}} \sqrt{a / \tau} / 2$ is the normalized admission probability of a candidate of type $b^{\prime}=l, r$ in state $i$.

In the unanimity voting case, we have

$$
\gamma^{u} \equiv 4 q_{3} /\left(y_{3}^{r}+y_{3}^{l}\right)+q_{2}\left(1+3\left(y_{1}^{l}\right)^{2}+3\left(y_{2}^{l}\right)^{2}\right) /\left(y_{1}^{l}+y_{2}^{l}\right) .
$$

From the above expressions, we can calculate the expected welfare loss in each case and obtain the following result.

## PROPOSITION 5:

(i) For every $c$, the club can achieve greater or equal long-term welfare under unanimity voting than under majority voting.
(ii) Under majority voting, the club cannot achieve greater long-term welfare than it can in the harmonious equilibrium.


Figure 2. Long-Term Welfare Loss in the Most Efficient Equilibrium
(iii) For $c<0.42$, under unanimity voting the club can achieve higher long-term welfare than in the harmonious equilibrium. At $c=0.24$, the club achieves the greatest long-term welfare under unanimity voting.

Figure 2 illustrates the comparison of welfare losses $\gamma$ in different cases. Evidently, welfare loss is never lower under majority voting than under unanimity voting. Thus, Proposition 5(i) says that unanimity voting is better than majority voting when the club's admission of new members is influenced by internal politics. As can be seen in Figure 2, unanimity voting outperforms majority voting in two scenarios. First, when $c<0.43$, the pro-minority power-switching equilibrium under unanimity rule achieves greater long-term welfare than the pro-majority power-switching equilibrium under majority rule. As shown in Figure 1, candidates of both types face stringent admission standards in the pro-minority power-switching equilibrium under unanimity voting, while candidates of the majority type are admitted with a much lower standard in the pro-majority power-switching equilibrium under majority voting. As a result, by giving both types of incumbent members more balanced power in admitting new members, unanimity voting can avoid straightforward favoritism by the majority-type incumbents and motivate all members to search for high quality candidates. Secondly, for $c \in(0.43,1.97)$, the most efficient equilibrium under unanimity voting is the power-switching one, but majority rule only allows the glass-ceiling equilibrium, which is much less efficient. Intuitively, majority
voting gives the majority-type incumbent members unlimited power to exclude the opposite type candidates. Thus, as the stake of internal politics becomes large, they will simply set a glass ceiling to exclude the opposite type and keep control of the club firmly in their own hands.

Proposition 5(ii) says that majority voting always yields lower long-term welfare than the harmonious equilibrium. As shown in Corollary 1, in either the pro-majority power-switching or the glass-ceiling equilibrium, admission standards are biased relative to those in the harmonious equilibrium in that candidates of one type face a much lower standard than that applied to candidates of the other type. Thus, as the welfare loss function is convex in admission standards, the divergence of admission standards for the two types of candidates (relative to that in the harmonious equilibrium) leads to lower long-term welfare under majority voting than does the harmonious equilibrium.

As is also evident from Figure 2, Proposition 5(iii) says that for $c<0.42$, the pro-minority power-switching equilibrium under unanimity voting yields greater long-term welfare than does the harmonious equilibrium. The reason can be clearly seen in Figure 1, as the admission standards for both types of candidates are higher in the pro-minority power-switching equilibrium than in the harmonious equilibrium. When internal politics are mild, under unanimity voting both types of incumbent members will set stringent standards to admit candidates of the opposite type, which helps offset the intertemporal free-riding problem in the harmonious equilibrium.

We should point out that the superiority of unanimity voting in our model crucially depends on our focus on the most efficient equilibria. Unanimity voting is more likely to have multiple equilibria than majority voting because coordination between different members in the same generation and across generations is more important. Thus, whether a unanimity voting rule is good for the club depends on whether the club members can manage to select the best equilibrium. If they fail to do so, unanimity voting can lead to worse outcomes than majority voting. ${ }^{22}$

By showing that unanimity rule dominates majority rule, Proposition 5 provides a new rationale for unanimity voting. In the common-value voting literature, unanimity voting is found to be an inferior collective decision mechanism (see, e.g., Feddersen and Pesendorfer 1998). These different results should not be viewed as contradictory because the contexts are quite different. In our model, voting is used to aggregate preferences in a collective search situation, while the common-value voting literature considers information aggregation in collective decisions. ${ }^{23}$ Another common criticism of committee decision making is that it causes inefficiently long delays in reaching agreements, ${ }^{24}$ and clearly delays will be the longest under unanimity voting. Our analysis shows that in the presence of internal politics, unanimity

[^12]voting is effective in motivating members to engage in costly search and thus increases the club's long-term welfare. Indeed, it takes longer to reach a decision under unanimity voting than under majority voting in our model, but this is actually good for the organization (although not necessarily for individual members who have to incur personal delay costs). ${ }^{25}$

Our analysis suggests that organizations may benefit from requiring important decisions to be made by consensus. For example, university administrations quite often approve senior hiring proposals by academic departments only if those proposals have had super-majority or even unanimous support from the departments; a mere majority support is usually perceived as a weak signal by university administrations. Similarly, in many partnership firms, new partners may only be admitted by unanimous vote of the existing partners. A casual argument for such requirements is that even though decision-making processes may take a long time in organizations that emphasize consensus building, they tend to make better decisions as all members are involved in decision making and tend to be more balanced as no group of members can dominate by forming a majority coalition. Our analysis provides conditions under which such requirements are indeed optimal. ${ }^{26}$

Our results also suggest that there is an optimal degree of organizational incongruity. In a homogeneous organization, it is easy to reach agreements but members tend to shirk in their efforts at making important decisions for the organization due to the intertemporal free-riding problem. However, in a highly divided organization, internal politicking is so intense that decision-making processes are exceedingly long and costly, and the organization eventually becomes perpetually dominated by one type. A good organizational design should avoid these two extremes by trying to achieve the right degree of incongruity. ${ }^{27}$ In other words, internal politics, whereby members of an organization compete for discretionary rents, if designed properly, can be a useful incentive instrument. In such cases, the organization will remain balanced over time and members of different types will be engaged in making the important decisions of the organization, resulting in better decisions and better longterm outcomes for the organization. ${ }^{28}$

[^13]
## VI. Discussions and Concluding Remarks

In this paper, we build an infinite-horizon dynamic model to study the interactions between an organization's internal politics and its admission decisions. Among other things, we find that it is beneficial for organizations to build consensus in the presence of internal politics, that is, unanimity voting does a better job than majority voting in terms of long-term welfare. In addition, internal politics can be a useful incentive instrument in organizational design: organizations with a certain degree of incongruity perform better in the long run than either harmonious or very divided organizations under unanimity voting.

To simplify the welfare comparison, we consider a model with distributive politics, in which the total rent in each period is constant and is shared by the majority-type incumbents, so that the type profile of the club does not affect welfare directly. This naturally raises the concern about whether our main results are driven by rent dilution in homogeneous states. In the online Appendix, we study extensions where each incumbent majority member receives a fixed amount of rent. One such example is where the two most senior members share the rent in homogeneous states. This example also addresses the concern that within-type equal sharing rule is not incentive-compatible in homogeneous states because two members have incentives to form a coalition and exclude the other one from sharing the rent. We show that in these extensions, most of the findings regarding long-term welfare are similar to the baseline model.

In the online Appendix, we consider another extension where rent is distributed via Nash bargaining. In homogeneous states, each member has equal bargaining power $1 / 3$, implying that rent is shared equally among them; in contentious states, majority members have equal bargaining power $\nu \in(1 / 3,1 / 2]$, while the minority member has bargaining power $1-2 \nu$. We assume that $\nu>1 / 3$ to capture the benefits of internal politics. It turns out that this extension is exactly isomorphic to our baseline model if we redefine $c=\frac{\left(\nu-\frac{1}{3}\right) B}{2 \sqrt{a \tau}}$ (in our model, $\nu=1 / 2$ and this coincides with our expression of the degree of incongruity). Therefore, our main findings regarding long-term welfare are also robust to this extension. This extension has another interesting implication on organizational design: even when the parameters $a, \tau$, and $B$ are all exogenous, the organization can still achieve the right degree of incongruity by adjusting the value of $\nu$. In particular, Proposition 5(iii) implies that the optimal $\nu$ is $1 / 2$ when $\frac{B}{12 \sqrt{a \tau}}<0.24$, and is $\frac{0.48 \sqrt{a \tau}}{B}+\frac{1}{3}$ otherwise. These robustness analyses indicate that our main results are not driven by rent dilution in homogeneous states. Instead, they are mainly built on the fact that majority and minority members receive different amounts of rent in contentious states.

Our model can be extended in various directions. In the online Appendix, we present several extensions, including models with different quality distributions, different discounting factors, and a larger club size. The main results of the baseline model are mostly robust in these extensions. It would also be interesting to consider other extensions in future research, such as the case where candidates can endogenously choose qualities by making human capital investments as in Athey, Avery,
and Zemsky (2000) and Sobel (2000, 2001). Given the admission biases in the equilibria of the model, candidates of the two types will have different incentives to make human capital investments. An investigation of this setting may provide insight into when candidates of the types that are discriminated against will make greater (or smaller) human capital investments than candidates of the other type. Another possible extension would consider a model in which the benefits of club membership have two components, as in Esteban and Ray (2001). Besides the value that his quality provides to every member, a candidate may also bring an additional common value only to incumbent members of his type (e.g., a new theorist benefits incumbent theorists in a department). Finally, our model can be also extended to add other interesting factors such as competition among organizations for members as in Prüfer and Walz (2013) or learning about the members' preferences as in Strulovici (2010).

## Appendix

## A. Equilibrium Analysis under Majority Voting

Under majority voting, consider a right-type incumbent member A. The left-type incumbent's expected search payoff can be calculated similarly to that of member A. From Section IIB, we only need to solve A's search payoff where the subscript $i$ denotes the current state, the superscript $R$ denotes A's type, and the admission policy $\sigma$ is suppressed to simplify notation.

In state $i=2$, if the club admits a right-type candidate with quality $v^{r}$ in the first selection round, A's expected search payoff is
(A.1) $\pi_{2}^{R}\left(v^{r}\right.$, yes $)=\frac{2}{3}\left[v^{r}+\frac{1}{2}\left(\frac{B}{2}+\frac{1}{2} v^{r}+\pi_{2}^{R}\right)+\frac{1}{2}\left(\frac{B}{3}+\frac{1}{2} v^{r}+\pi_{3}^{R}\right)\right]$.

Equation (A.1) can be further simplified as

$$
\pi_{2}^{R}\left(v^{r}, y e s\right)=v^{r}+\frac{5 B}{18}+\frac{1}{3} \pi_{2}^{R}+\frac{1}{3} \pi_{3}^{R},
$$

which again implies that the expected value to incumbent member A if a candidate with quality $v^{r}$ is admitted is $v^{r}$.

Similarly, in state $i=2$, if a left-type candidate with quality $v^{l}$ is admitted, a right-type incumbent member A's expected $\pi_{2}^{R}$ is

$$
\begin{equation*}
\pi_{2}^{R}\left(v^{l}, \text { yes }\right)=v^{l}+\frac{1}{3} \pi_{1}^{R}+\frac{1}{3}\left(\frac{B}{2}+\pi_{2}^{R}\right) . \tag{A.2}
\end{equation*}
$$

In state $i=3$, member A's expected search payoff from admitting a left-type candidate with quality $v^{l}$ or a right-type candidate with quality $v^{r}$ can be calculated as follows:

$$
\begin{equation*}
\pi_{3}^{R}\left(v^{l}, y e s\right)=v^{l}+\frac{B}{3}+\frac{2}{3} \pi_{2}^{R}, \quad \pi_{3}^{R}\left(v^{r}, y e s\right)=v^{r}+\frac{2 B}{9}+\frac{2}{3} \pi_{3}^{R} . \tag{A.3}
\end{equation*}
$$

If a candidate is rejected by the club, no matter what the type or quality of the candidate is, a type $b \in\{L, R\}$ incumbent member's search payoff simply becomes $\pi_{i}^{b}-\tau$.

Notice that under majority voting, the block of right-type incumbents decide the admission policy $\left(v_{i}^{l}, v_{i}^{r}\right)$ in state $i=2,3$. As a result, equation (5) implies:

$$
\begin{equation*}
\pi_{i}^{R}=E\left[\frac{1}{2} \max \left\{\pi_{i}^{R}\left(v^{r}, y e s\right), \pi_{i}^{R}-\tau\right\}+\frac{1}{2} \max \left\{\pi_{i}^{R}\left(v^{l}, \text { yes }\right), \pi_{i}^{R}-\tau\right\}\right] \tag{A.4}
\end{equation*}
$$

where $\pi_{i}^{R}(\cdot$, yes) is defined by equations (A.1) through (A.3).
Given the equilibrium admission policy $\left(v_{i}^{l}, v_{i}^{r}\right)$, we can now calculate the expected search payoff of a type $b \in\{R, L\}$ incumbent member in state $i$ as follows:

$$
\begin{align*}
\pi_{i}^{b}=0.5\left[\int_{v_{i}^{r}}^{\bar{v}}\right. & \pi_{i}^{b}\left(v_{i}^{r}, \text { yes }\right) d F\left(v_{i}^{r}\right)+F\left(v_{i}^{r}\right)\left(\pi_{i}^{b}-\tau\right)  \tag{A.5}\\
& \left.+\int_{v_{i}^{l}}^{\bar{v}} \pi_{i}^{b}\left(v_{i}^{l}, \text { yes }\right) d F\left(v_{i}^{l}\right)+F\left(v_{i}^{l}\right)\left(\pi_{i}^{b}-\tau\right)\right] .
\end{align*}
$$

PROOF OF PROPOSITION 3 STEP (i): Characterizing the Power-Switching Equilibrium

Under majority voting rule, the first possibility is that both types of candidates are admitted in state 2 . Then condition (4) is satisfied with equality for $i=2,3$ and $b^{\prime}=l, r$. After some algebra calculation, we have

$$
\begin{equation*}
\frac{2}{3}\left[\frac{3}{2} v_{3}^{r}+\frac{B}{3}+\pi_{3}^{R}\right]=\pi_{3}^{R}-\tau \tag{A.6}
\end{equation*}
$$

$$
\begin{equation*}
\frac{2}{3}\left[\frac{3}{2} v_{3}^{l}+\frac{B}{2}+\pi_{2}^{R}\right]=\pi_{3}^{R}-\tau \tag{A.7}
\end{equation*}
$$

$$
\begin{equation*}
\frac{2}{3}\left[\frac{3}{2} v_{2}^{r}+\frac{5 B}{12}+\frac{1}{2} \pi_{2}^{R}+\frac{1}{2} \pi_{3}^{R}\right]=\pi_{2}^{R}-\tau \tag{A.8}
\end{equation*}
$$

$$
\begin{equation*}
\frac{2}{3}\left[\frac{3}{2} v_{2}^{l}+\frac{B}{4}+\frac{1}{2} \pi_{1}^{R}+\frac{1}{2} \pi_{2}^{R}\right]=\pi_{2}^{R}-\tau \tag{A.9}
\end{equation*}
$$

By equation (A.5), we can obtain, for $i=2,3$ and $b=r$,
(A.10)

$$
\begin{aligned}
\pi_{3}^{R}= & \frac{\bar{v}-v_{3}^{r}}{3 a}\left[\frac{B}{3}+\pi_{3}^{R}\right]+\frac{\left[\bar{v}^{2}-\left(v_{3}^{r}\right)^{2}\right]}{4 a}+\frac{v_{3}^{r}-\underline{v}}{2 a}\left[\pi_{3}^{R}-\tau\right] \\
& +\frac{\bar{v}-v_{3}^{l}}{3 a}\left[\frac{B}{2}+\pi_{2}^{R}\right]+\frac{\left[\bar{v}^{2}-\left(v_{3}^{l}\right)^{2}\right]}{4 a}+\frac{v_{3}^{l}-\underline{v}}{2 a}\left[\pi_{3}^{R}-\tau\right]
\end{aligned}
$$

(A.11) $\pi_{2}^{R}=\frac{\bar{v}-v_{2}^{r}}{3 a}\left[\frac{5 B}{12}+\frac{1}{2} \pi_{2}^{R}+\frac{1}{2} \pi_{3}^{R}\right]+\frac{\left[\bar{v}^{2}-\left(v_{2}^{r}\right)^{2}\right]}{4 a}+\frac{v_{2}^{r}-\underline{v}}{2 a}\left[\pi_{2}^{R}-\tau\right]$

$$
+\frac{\bar{v}-v_{2}^{l}}{3 a}\left[\frac{B}{4}+\frac{1}{2} \pi_{1}^{R}+\frac{1}{2} \pi_{2}^{R}\right]+\frac{\left[\bar{v}^{2}-\left(v_{2}^{l}\right)^{2}\right]}{4 a}+\frac{v_{2}^{l}-\underline{v}}{2 a}\left[\pi_{2}^{R}-\tau\right]
$$

Also by equation (A.5), and using the fact that $\pi_{2}^{L}=\pi_{1}^{R}$, we have
(A.12) $\pi_{1}^{R}=\frac{\bar{v}-v_{2}^{r}}{3 a} \pi_{1}^{R}+\frac{\left[\bar{v}^{2}-\left(v_{2}^{r}\right)^{2}\right]}{4 a}+\frac{v_{2}^{r}-\underline{v}}{2 a}\left[\pi_{1}^{R}-\tau\right]$

$$
+\frac{\bar{v}-v_{2}^{l}}{3 a}\left[\frac{B}{2}+\pi_{2}^{R}\right]+\frac{\left[\bar{v}^{2}-\left(v_{2}^{l}\right)^{2}\right]}{4 a}+\frac{v_{2}^{l}-\underline{v}}{2 a}\left[\pi_{1}^{R}-\tau\right] .
$$

Thus, we have a system of seven equations (A.6)-(A.12) with seven unknowns:

$$
v_{2}^{r}, v_{2}^{l}, v_{3}^{r}, v_{3}^{l}, \pi_{1}^{R}, \pi_{2}^{R}, \pi_{3}^{R}
$$

Substituting (A.6) and (A.7) into (A.10) and simplifying, we can get $4 a \tau$ $=\left(\bar{v}-v_{3}^{r}\right)^{2}+\left(\bar{v}-v_{3}^{l}\right)^{2}$. Using our variable transformation $x_{i}^{b^{\prime}} \equiv\left(\bar{v}-v_{i}^{b^{\prime}}\right) / a$, we have

$$
\begin{equation*}
\left(x_{3}^{r}\right)^{2}+\left(x_{3}^{l}\right)^{2}=4 \tau / a \tag{A.13}
\end{equation*}
$$

Similarly, substituting equations (A.8) and (A.9) into (A.11) and simplifying, we can get $4 a \tau=\left(\bar{v}-v_{2}^{r}\right)^{2}+\left(\bar{v}-v_{2}^{l}\right)^{2}$, or

$$
\begin{equation*}
\left(x_{2}^{r}\right)^{2}+\left(x_{2}^{l}\right)^{2}=4 \tau / a \tag{A.14}
\end{equation*}
$$

From equations (A.6), (A.7), and (A.9), we can get

$$
\begin{aligned}
& \pi_{3}^{R}=\frac{2 B}{3}+3 \tau+3 v_{3}^{r} \\
& \pi_{2}^{R}=\frac{B}{2}+3 \tau+\frac{9}{2} v_{3}^{r}-\frac{3}{2} v_{3}^{l} \\
& \pi_{1}^{R}=\frac{B}{2}+3 \tau+9 v_{3}^{r}-3 v_{3}^{l}-3 v_{2}^{l} .
\end{aligned}
$$

Substituting $\pi_{3}^{R}$ and $\pi_{2}^{R}$ into (A.8) gives $B / 6=2 v_{3}^{r}-v_{3}^{l}-v_{2}^{r}=\left(\bar{v}-v_{2}^{r}\right)+$ $\left(\bar{v}-v_{3}^{l}\right)-2\left(\bar{v}-v_{3}^{r}\right)$. Thus,

$$
\begin{equation*}
x_{2}^{r}+x_{3}^{l}-2 x_{3}^{r}=\frac{B}{6 a} . \tag{A.15}
\end{equation*}
$$

Substituting $\pi_{1}^{R}, \pi_{2}^{R}$ into (A.12) and manipulating terms, we can obtain

$$
\begin{equation*}
\left(x_{2}^{r}+x_{2}^{l}\right) B / a=3 x_{2}^{r} x_{3}^{r}+3 x_{2}^{l} x_{3}^{l}+6 x_{2}^{l} x_{2}^{r}-12\left(x_{2}^{l}\right)^{2} . \tag{A.16}
\end{equation*}
$$

Thus, we have four equations (A.13)-(A.16) and four unknowns: $x_{3}^{r}, x_{3}^{l}, x_{2}^{r}$, and $x_{2}^{l}$. To further simplify things, let $y_{i}^{b^{\prime}}=x_{i}^{b^{\prime}} \sqrt{a / \tau} / 2$, for $i=1,2,3,4$ and $b^{\prime}=l$, $r$. Define $c=B /(12 \sqrt{a \tau})$. Then (A.13)-(A.16) become

$$
\begin{align*}
\left(y_{3}^{r}\right)^{2}+\left(y_{3}^{l}\right)^{2} & =1  \tag{A.17}\\
\left(y_{2}^{r}\right)^{2}+\left(y_{2}^{l}\right)^{2} & =1 \\
y_{2}^{r}+y_{3}^{l}-2 y_{3}^{r} & =c \\
y_{2}^{r} y_{3}^{r}+y_{2}^{l} y_{3}^{l}+2 y_{2}^{l} y_{2}^{r}-4\left(y_{2}^{l}\right)^{2} & =2 c\left(y_{2}^{r}+y_{2}^{l}\right)
\end{align*}
$$

By the first two equations of (A.17), all $y_{i}^{b^{\prime}}$ must be in $(0,1)$. A solution to (A.17) must also have the following properties:

CLAIM 1: If $c=0$, then $y_{i}^{b^{\prime}}=\sqrt{2} / 2$ is a solution that coincides with the harmonious equilibrium.

## PROOF:

It is easy to check that $y_{i}^{b^{\prime}}=\sqrt{2} / 2$ is a solution to (A.17) when $c=0$. Then $x_{i}^{b^{\prime}}=2 y_{i}^{b^{\prime}} \sqrt{\tau / a}=\sqrt{2 \tau / a}$. By our calculation in Section III, in the harmonious equilibrium, $\hat{x}=(\bar{v}-\hat{v}) / a=\sqrt{2 \tau / a}$.

CLAIM 2: $y_{2}^{l}$ cannot be the largest among the four unknowns. Otherwise, the righthand (RHS) side of the last equation of (A.17) is negative. Contradiction.

CLAIM 3: $y_{3}^{r} \leq y_{3}^{l}$.
PROOF:
Otherwise, if $y_{3}^{r}>y_{3}^{l}$, the third equation of (A.17) implies that

$$
y_{2}^{r}=2 y_{3}^{r}+c-y_{3}^{l}>y_{3}^{l} .
$$

Then it must be that $y_{2}^{r}>y_{3}^{r}>y_{3}^{l}>y_{2}^{l}$, where the last inequality follows from $\left(y_{3}^{r}\right)^{2}+\left(y_{3}^{l}\right)^{2}=\left(y_{2}^{r}\right)^{2}+\left(y_{2}^{l}\right)^{2}$. However, substituting the third equation (as the expression of $c$ ) into the last equation of (A.17) gives

$$
2\left(y_{2}^{r}\right)^{2}+2 y_{2}^{r} y_{3}^{l}+y_{3}^{l} y_{2}^{l}-5 y_{2}^{r} y_{3}^{r}+4\left(y_{2}^{l}\right)^{2}-4 y_{3}^{r} y_{2}^{l}=0
$$

This is inconsistent with the fact that $y_{2}^{r}$ and $y_{3}^{r}$ are the largest. Contradiction.
CLAIM 4: $y_{3}^{r} \leq y_{2}^{r}$. Otherwise, it must be that $y_{2}^{r}<y_{3}^{r} \leq y_{3}^{l}<y_{2}^{l}$, since $\left(y_{3}^{r}\right)^{2}+$ $\left(y_{3}^{l}\right)^{2}=\left(y_{2}^{r}\right)^{2}+\left(y_{2}^{l}\right)^{2}$. But this violates Claim 2. Contradiction.

CLAIM 5: $y_{2}^{r} \geq y_{3}^{l} \geq \sqrt{2} / 2 \geq y_{3}^{r} \geq y_{2}^{l}$.
PROOF:
Suppose $y_{3}^{l}>y_{2}^{r}$. Then it must be that $y_{3}^{l}>\left\{y_{2}^{r}, y_{2}^{l}\right\} \geq y_{3}^{r}$. From the third equation of (A.17), $y_{2}^{r}=2 y_{3}^{r}+c-y_{3}^{l}$. Substituting this into the third term of the left-hand side (LHS) of the last equation of (A.17), we have

$$
y_{2}^{r} y_{3}^{r}-y_{2}^{l} y_{3}^{l}+4 y_{2}^{l} y_{3}^{r}-4\left(y_{2}^{l}\right)^{2}=2 c y_{2}^{r} .
$$

The LHS is negative when $y_{3}^{l}>\left\{y_{2}^{r}, y_{2}^{l}\right\} \geq y_{3}^{r}$, because $4 y_{2}^{l} y_{3}^{r} \leq 4\left(y_{2}^{l}\right)^{2}$ and $y_{2}^{r} y_{3}^{r}<y_{2}^{l} y_{3}^{l}$. Therefore, it must be that $y_{2}^{r} \geq y_{3}^{l}$. By Claims 4 and 5 and the fact that $\left(y_{3}^{r}\right)^{2}+\left(y_{3}^{l}\right)^{2}=\left(y_{2}^{r}\right)^{2}+\left(y_{2}^{l}\right)^{2}=1$, it must be that $y_{2}^{r} \geq y_{3}^{l} \geq \sqrt{2} / 2 \geq$ $y_{3}^{r} \geq y_{2}^{l}$.

Substituting the first two equations of (A.17) into the last two gives

$$
\begin{aligned}
& y_{2}^{r}+\sqrt{1-\left(y_{3}^{r}\right)^{2}}-2 y_{3}^{r}=c \\
& y_{2}^{r} y_{3}^{r}+\sqrt{1-\left(y_{2}^{r}\right)^{2}} \sqrt{1-\left(y_{3}^{r}\right)^{2}}+2 y_{2}^{r} \sqrt{1-\left(y_{2}^{r}\right)^{2}}-4\left(1-\left(y_{2}^{r}\right)^{2}\right) \\
&=2 c\left(y_{2}^{r}+\sqrt{1-\left(y_{2}^{r}\right)^{2}}\right)
\end{aligned}
$$

Substituting the first equation above into the second gives one equation in terms of $y_{3}^{r}$ only: $\Omega\left(y_{3}^{r} ; c\right)=0$, where function $\Omega$ is defined as follows:

$$
\begin{aligned}
\Omega(y ; c)= & \left(c+2 y-\sqrt{1-y^{2}}\right) y+\sqrt{1-\left(c+2 y-\sqrt{1-y^{2}}\right)^{2}} \sqrt{1-y^{2}} \\
& +2\left(c+2 y-\sqrt{1-y^{2}}\right) \sqrt{1-\left(c+2 y-\sqrt{1-y^{2}}\right)^{2}} \\
& -4\left(1-\left(c+2 y-\sqrt{1-y^{2}}\right)^{2}\right) \\
& -2 c\left[\left(c+2 y-\sqrt{1-y^{2}}\right)+\sqrt{1-\left(c+2 y-\sqrt{1-y^{2}}\right)^{2}}\right] .
\end{aligned}
$$

If the power-switching equilibrium exists, we must have $\Omega(y ; c)=0$ for some $y \in \Phi=[\underline{y}, \sqrt{2} / 2]$, where $\underline{y}$ satisfies $c+2 y-\sqrt{1-y^{2}}=\sqrt{2} / 2$. Notice that when $y=\underline{y}, \Omega(y ; c)<0$. Therefore, there is a solution to equation $\Omega(y ; c)=0$ only when $\max _{y \in \Phi} \Omega(y ; c)>0$. It can be shown numerically that $\max _{y \in \Phi} \Omega(y ; c)>0$ for $c<0.43$ and vice versa. Therefore, the power-switching equilibrium exists for $c<0.43$.

PROOF OF PROPOSITION 3 STEP (ii): Characterizing the Glass-Ceiling Equilibrium

Another possibility is that $v_{2}^{l}=\bar{v}$, and hence by condition (4), the following condition must hold:

$$
\begin{equation*}
\frac{2}{3}\left[\frac{3}{2} \bar{v}+\frac{1}{2} \pi_{1}^{R}+\frac{1}{2} \pi_{2}^{R}+\frac{B}{4}\right] \leq \pi_{2}^{R}-\tau \tag{A.18}
\end{equation*}
$$

With $v_{2}^{l}=\bar{v}$, equations (A.6), (A.7), (A.8), and (A.10) should still hold and equations (A.11) and (A.12) are changed to

$$
\begin{equation*}
\pi_{2}^{R}=\frac{\pi_{2}^{R}-\tau}{2}+\frac{\bar{v}-v_{2}^{r}}{3 a}\left[\frac{5 B}{12}+\frac{1}{2} \pi_{2}^{R}+\frac{1}{2} \pi_{3}^{R}\right]+\frac{\left[\bar{v}^{2}-\left(v_{2}^{r}\right)^{2}\right]}{4 a}+\frac{v_{2}^{r}-\underline{v}}{2 a}\left[\pi_{2}^{R}-\tau\right] \tag{A.19}
\end{equation*}
$$

$$
\begin{equation*}
\pi_{1}^{R}=\frac{\pi_{1}^{R}-\tau}{2}+\frac{\bar{v}-v_{2}^{r}}{3 a} \pi_{1}^{R}+\frac{\left[\bar{v}^{2}-\left(v_{2}^{r}\right)^{2}\right]}{4 a}+\frac{v_{2}^{r}-\underline{v}}{2 a}\left[\pi_{1}^{R}-\tau\right] \tag{A.20}
\end{equation*}
$$

Thus, we have six equations (A.6), (A.7), (A.8), (A.10), (A.19), and (A.20) with six unknowns: $v_{2}^{r}, v_{3}^{r}, v_{3}^{l}, \pi_{1}^{R}, \pi_{2}^{R}, \pi_{3}^{R}$. The solution to this equation system must also satisfy (A.18) for it to constitute an equilibrium.

Similarly, define $x_{i}^{b^{\prime}} \equiv\left(\bar{v}-v_{i}^{b^{\prime}}\right) / a$ and $y_{i}^{b^{\prime}}=x_{i}^{b^{\prime}} \sqrt{a / \tau} / 2$. From the first and third equations of (A.17), $\left(y_{3}^{r}\right)^{2}+\left(y_{3}^{l}\right)^{2}=1$ and $y_{3}^{l}-2 y_{3}^{r}=c-1$. We can obtain the following solution:

$$
\begin{aligned}
& y_{3}^{r}=\frac{1}{5}\left(\sqrt{4+2 c-c^{2}}-2 c+2\right) \\
& y_{3}^{l}=\frac{1}{5}\left(2 \sqrt{4+2 c-c^{2}}+c-1\right)
\end{aligned}
$$

Then from equations (A.19) and (A.20), we can get

$$
\begin{aligned}
& \pi_{3}^{R}=\frac{2 B}{3}+3 \tau+3 v_{3}^{r} \\
& \pi_{2}^{R}=3 \bar{v}+3 \tau+\frac{3}{4} B-3 \sqrt{a \tau}\left(1+y_{3}^{r}\right) \\
& \pi_{1}^{R}=3 \bar{v}+3 \tau-6 \sqrt{a \tau} .
\end{aligned}
$$

Substituting $\pi_{2}^{R}$ and $\pi_{1}^{R}$ into (A.18), we get $y_{3}^{r} \leq 2 c$. A glass-ceiling equilibrium exists only if $y_{3}^{r}$ is not too large. This guarantees that $\pi_{3}^{R}$ is high enough to ensure that the majority members in contentious states always have incentives to wait for candidates with their own type other than accepting a candidate with the opposite type. Since $y_{3}^{r}$ is shown to be $\frac{1}{5}\left(\sqrt{4+2 c-c^{2}}-2 c+2\right)$, the condition $y_{3}^{r} \leq 2 c$ is satisfied if and only if $c \geq 10 / 29$. Therefore, a glass-ceiling equilibrium exists when $c \geq 10 / 29$. Also notice that when $c>2$, it's not difficult to verify that $\left\{y_{3}^{r}=0, y_{3}^{l}=1\right\}$ constitutes a glass-ceiling equilibrium, even though $\frac{1}{5}\left(\sqrt{4+2 c-c^{2}}-2 c+2\right)<0$.

PROOF OF PROPOSITION 3 STEP (iii): Comparing Long-Term Welfare
Under majority voting, using equation (A.5) in the online Appendix, we can show that the long-term welfare of the club is given by

$$
U^{m}=3 E v+\frac{3}{2} a+\tau-2 \sqrt{a \tau} \gamma^{m}
$$

where $\gamma^{m} \equiv 4 q_{3} /\left(y_{3}^{r}+y_{3}^{l}\right)+4 q_{2} /\left(y_{2}^{r}+y_{2}^{l}\right)$. For $c \in[10 / 29,0.43)$, both the power-switching and glass-ceiling equilibria exist. But the power-switching equilibrium yields greater long-term welfare than does the glass-ceiling equilibrium. This completes the proof of the proposition.

## PROOF OF COROLLARY 1:

From Claim 5 in Step (i) of the proof of Proposition 3, we immediately see that in any power-switching equilibrium, the majority-type incumbents in contentious states favor candidates of their own type and discriminate against candidates
of the opposite type: $x_{2}^{r}>\hat{x}=\sqrt{2 \tau / a}>x_{2}^{l}$; but in homogeneous states, they have lower standards for the opposite type than for their own type: $x_{3}^{l}>\hat{x}$ $=\sqrt{2 \tau / a}>x_{3}^{r}$. Since $y_{2}^{r} \geq y_{3}^{l} \geq \sqrt{2} / 2 \geq y_{3}^{r} \geq y_{2}^{l}$, the distortions are greater in contentious states than in homogeneous states: $x_{2}^{r}-x_{2}^{l} \geq x_{3}^{l}-x_{3}^{r}$.

Now consider a glass-ceiling equilibrium, which exists only when $c \geq 10 / 29$. It is trivial to see that $x_{2}^{r}=1>\hat{x}=\sqrt{2 \tau / a}>x_{2}^{l}=0$. In homogeneous states, when $c=10 / 29, y_{3}^{r}=2 c<\sqrt{2} / 2<y_{3}^{l}$. It is straightforward to check that when $c>10 / 29, y_{3}^{l}>y_{3}^{r}$ since $y_{3}^{l}$ is increasing in $c$ and $y_{3}^{r}$ is decreasing in $c$. Hence, we also get $x_{3}^{l}>\hat{x}=\sqrt{2 \tau / a}>x_{3}^{r}$.

## B. Equilibrium Analysis under Unanimity Voting

Abusing notation slightly, let $v_{2}^{r}\left(v_{1}^{r}\right)$ and $v_{2}^{l}\left(v_{1}^{l}\right)$ be the right-type incumbents' preferred quality standards in state 2 (1) for right- and left-type candidates, respectively. By symmetry, $v_{1}^{r}$ (respectively, $v_{1}^{l}$ ) is the left-type incumbent's preferred standard for a left-type (respectively, right-type) candidate in state 2 . Then, the admission criterion in state 2 is $\tilde{v}_{2}^{r}=\max \left\{v_{2}^{r}, v_{1}^{l}\right\}$ for right-type candidates, and $\tilde{v}_{2}^{l}=\max \left\{v_{2}^{l}, v_{1}^{r}\right\}$ for left-type candidates. Lemma A. 1 below says that under unanimity voting, the admission criterion for a candidate is determined by the preferred standard of the incumbent members of the opposite type.

LEMMA A.1: Under unanimity voting rule, in any equilibrium $v_{2}^{r} \leq v_{1}^{l}$ and $v_{2}^{l} \geq v_{1}^{r}$. Thus, $\tilde{v}_{2}^{r}=v_{1}^{l}$ and $\tilde{v}_{2}^{l}=v_{2}^{l}$.

## PROOF OF LEMMA A.1:

First we can show the following result.
LEMMA A.2:

$$
\begin{aligned}
& \left(x_{2}^{r}\right)^{2}+\left(x_{2}^{l}\right)^{2}=\frac{4 \tau}{a}+\left[\max \left\{x_{2}^{r}, x_{1}^{l}\right\}-x_{1}^{l}\right]^{2}+\left[\max \left\{x_{2}^{l}, x_{1}^{r}\right\}-x_{1}^{r}\right]^{2} \\
& \left(x_{1}^{r}\right)^{2}+\left(x_{1}^{l}\right)^{2}=\frac{4 \tau}{a}+\left[\max \left\{x_{2}^{r}, x_{1}^{l}\right\}-x_{2}^{r}\right]^{2}+\left[\max \left\{x_{2}^{l}, x_{1}^{r}\right\}-x_{2}^{l}\right]^{2} .
\end{aligned}
$$

## PROOF:

Since the admission criterion is now given by $\tilde{v}_{2}^{r}=\max \left\{v_{2}^{r}, v_{1}^{l}\right\}$ and $\tilde{v}_{2}^{l}=\max \left\{v_{2}^{l}, v_{1}^{r}\right\}$, equations (A.11) should be modified as follows:

$$
\begin{aligned}
\pi_{2}^{R}= & \frac{\bar{v}-\tilde{v}_{2}^{r}}{3 a}\left[\frac{5 B}{12}+\frac{1}{2} \pi_{2}^{R}+\frac{1}{2} \pi_{3}^{R}\right]+\frac{\left[\bar{v}^{2}-\left(\tilde{v}_{2}^{r}\right)^{2}\right]}{4 a}+\frac{\tilde{v}_{2}^{r}-\underline{v}}{2 a}\left[\pi_{2}^{R}-\tau\right] \\
& +\frac{\bar{v}-\tilde{v}_{2}^{l}}{3 a}\left[\frac{B}{4}+\frac{1}{2} \pi_{1}^{R}+\frac{1}{2} \pi_{2}^{R}\right]+\frac{\left[\bar{v}^{2}-\left(\tilde{v}_{2}^{l}\right)^{2}\right]}{4 a}+\frac{\tilde{v}_{2}^{l}-\underline{v}}{2 a}\left[\pi_{2}^{R}-\tau\right] .
\end{aligned}
$$

Moreover, equations (A.8) and (A.9) should also be satisfied by the requirement of sincere voting. Using equations (A.8) and (A.9), we can simplify the above equation as

$$
\left(\bar{v}-v_{2}^{r}\right)^{2}+\left(\bar{v}-v_{2}^{l}\right)^{2}=4 a \tau+\left(\tilde{v}_{2}^{r}-v_{2}^{r}\right)^{2}+\left(\tilde{v}_{2}^{l}-v_{2}^{l}\right)^{2}
$$

Since

$$
\frac{\tilde{v}_{2}^{r}-v_{2}^{r}}{a}=\frac{\bar{v}-v_{2}^{r}}{a}-\frac{\bar{v}-\tilde{v}_{2}^{r}}{a}=x_{2}^{r}-\min \left\{x_{2}^{r}, x_{1}^{l}\right\}=\max \left\{x_{2}^{r}, x_{1}^{l}\right\}-x_{1}^{l}
$$

and similarly

$$
\frac{v_{1}^{l}-v_{2}^{l}}{a}=\max \left\{x_{2}^{l}, x_{1}^{r}\right\}-x_{1}^{r}
$$

we get the first statement of the lemma. Using the same method on equation (A.12), and given that

$$
\begin{equation*}
\frac{2}{3}\left[\frac{3}{2} v_{1}^{r}+\frac{B}{2}+\pi_{2}^{R}\right]=\pi_{1}^{R}-\tau \tag{A.21}
\end{equation*}
$$

$$
\begin{equation*}
\frac{2}{3}\left[\frac{3}{2} v_{1}^{l}+\pi_{1}^{R}\right]=\pi_{1}^{R}-\tau \tag{A.22}
\end{equation*}
$$

we can prove the second statement of the lemma.
To prove the proposition, let's first consider an interior equilibrium where all the quality standards are less than $\bar{v}$. From equations (A.6)-(A.9) and (A.21)-(A.22), we can eliminate all the $\pi \mathrm{s}$ to get

$$
\begin{equation*}
x_{2}^{r}+x_{3}^{l}-2 x_{3}^{r}=\frac{B}{6 a} \tag{A.23}
\end{equation*}
$$

$$
\begin{equation*}
x_{3}^{r}-x_{2}^{l}+x_{2}^{r}-x_{1}^{l}=\frac{B}{3 a} \tag{A.24}
\end{equation*}
$$

$$
\begin{equation*}
x_{2}^{l}+x_{1}^{r}-2 x_{1}^{l}=\frac{B}{2 a} \tag{A.25}
\end{equation*}
$$

We now eliminate all the other possibilities to prove the proposition:
(a) Suppose $v_{1}^{l} \geq v_{2}^{r}, v_{1}^{r}>v_{2}^{l}$, then $x_{1}^{l} \leq x_{2}^{r}, x_{1}^{r}<x_{2}^{l}$. By the above lemma, we have

$$
\begin{aligned}
& \left(x_{2}^{r}\right)^{2}+\left(x_{2}^{l}\right)^{2}=\frac{4 \tau}{a}+\left(x_{2}^{r}-x_{1}^{l}\right)^{2}+\left(x_{2}^{l}-x_{1}^{r}\right)^{2} \\
& \left(x_{1}^{r}\right)^{2}+\left(x_{1}^{l}\right)^{2}=\frac{4 \tau}{a}
\end{aligned}
$$

Substituting the second equation into the first equation, we can get $x_{2}^{l} x_{1}^{r}+$ $x_{2}^{r} x_{1}^{l}=\left(x_{1}^{r}\right)^{2}+\left(x_{1}^{l}\right)^{2}$. But this cannot hold, because by $x_{1}^{l} \leq x_{2}^{r}$ and $x_{1}^{r}<$ $x_{2}^{l}$, the RHS is less than the LHS.
(b) Suppose $v_{1}^{l}<v_{2}^{r}, v_{1}^{r} \leq v_{2}^{l}$, then $x_{1}^{l}>x_{2}^{r}, x_{1}^{r} \geq x_{2}^{l}$. Following the same method used in part $(\mathrm{a})$, we can get $x_{2}^{r} x_{1}^{l}+x_{2}^{l} x_{1}^{r}=\left(x_{2}^{r}\right)^{2}+\left(x_{2}^{l}\right)^{2}$, which is impossible since $x_{1}^{l}>x_{2}^{r}, x_{1}^{r} \geq x_{2}^{l}$.
(c) Suppose $v_{1}^{l}<v_{2}^{r}, v_{1}^{r}>v_{2}^{l}$, then $x_{1}^{l}>x_{2}^{r}, x_{1}^{r}<x_{2}^{l}$. Equation (A.25) and $x_{1}^{r}<x_{2}^{l}$ imply that $x_{1}^{l}<x_{2}^{l}$. Equation (A.24) and $x_{1}^{l}>x_{2}^{r}$ imply that $x_{2}^{l}<$ $x_{3}^{r}$. Thus, we have $x_{2}^{r}<x_{1}^{l}<x_{2}^{l}<x_{3}^{r}$. By equation (A.23), we must have $x_{3}^{l}>x_{3}^{r}$. From Lemma A. 2 we have

$$
\begin{aligned}
& \left(x_{2}^{r}\right)^{2}+\left(x_{2}^{l}\right)^{2}=\frac{4 \tau}{a}+\left(x_{2}^{l}-x_{1}^{r}\right)^{2} \\
& \left(x_{1}^{r}\right)^{2}+\left(x_{1}^{l}\right)^{2}=\frac{4 \tau}{a}+\left(x_{1}^{l}-x_{2}^{r}\right)^{2} .
\end{aligned}
$$

Summing them up and substituting $\left(x_{3}^{r}\right)^{2}+\left(x_{3}^{l}\right)^{2}$ for $4 \tau / a$ (since equation (A.13) is still valid), we can get

$$
\begin{equation*}
\left(x_{3}^{r}\right)^{2}+\left(x_{3}^{l}\right)^{2}=x_{1}^{r} x_{2}^{l}+x_{2}^{r} x_{1}^{l} \tag{A.26}
\end{equation*}
$$

But this contradicts the fact that $x_{3}^{l}$ and $x_{3}^{r}$ are greater than all the four variables on the RHS.

In summary, in an interior equilibrium, it must be that $v_{1}^{l} \geq v_{2}^{r}$ and $v_{1}^{r} \leq v_{2}^{l}$.
Now consider that some of the standards are greater than $\bar{v}$. Parts (a) and (b) of the above proof are still valid. For part (c), assuming $\hat{v}_{3}^{r}$ satisfies equation (A.6), which means

$$
\frac{2}{3}\left[\frac{3}{2} \hat{v}_{3}^{r}+\frac{B}{3}+\pi_{3}^{R}\right]=\pi_{3}^{R}-\tau
$$

and assuming the same thing for equations (A.7)-(A.9) and (A.21)-(A.22), we can get $\hat{v}_{3}^{l}, \hat{v}_{2}^{r}, \hat{v}_{2}^{l}, \hat{v}_{1}^{r}, \hat{v}_{1}^{l}$, respectively. It's obvious that $v_{i}^{b^{\prime}}=\min \left\{\hat{v}_{i}^{b^{\prime}}, \bar{v}\right\}$.

Define $\hat{x}_{i}^{b^{\prime}} \equiv \frac{\bar{v}-\hat{v}_{i}^{b^{\prime}}}{a}$, then $x_{i}^{b^{\prime}}=\max \left\{\hat{x}_{i}^{b^{\prime}}, 0\right\}$ and (A.23)-(A.25) become

$$
\begin{align*}
\hat{x}_{2}^{r}+\hat{x}_{3}^{l}-2 \hat{x}_{3}^{r} & =\frac{B}{6 a} ;  \tag{A.27}\\
\hat{x}_{3}^{r}-\hat{x}_{2}^{l}+\hat{x}_{2}^{r}-\hat{x}_{1}^{l} & =\frac{B}{3 a}  \tag{A.28}\\
\hat{x}_{2}^{l}+\hat{x}_{1}^{r}-2 \hat{x}_{1}^{l} & =\frac{B}{2 a} . \tag{A.29}
\end{align*}
$$

Since $x_{1}^{l}>x_{2}^{r}, x_{1}^{r}<x_{2}^{l}$, it's straightforward that $\hat{x}_{1}^{l}>\hat{x}_{2}^{r}, \hat{x}_{1}^{r}<\hat{x}_{2}^{l}$. So we can follow the same analysis used in part (c) above to get that $\hat{x}_{3}^{r}$ and $\hat{x}_{3}^{l}$ are greater
than the other four $\hat{x}_{i}^{b^{\prime}}$. Noting that at least one $\hat{x}_{2}^{b^{\prime}}$ should be positive (otherwise in state 2 the club will not hire any candidate and will receive an expected utility of $-\infty)$, we see that $\hat{x}_{3}^{r}$ and $\hat{x}_{3}^{l}$ must be positive. Then we have $x_{3}^{r}=\hat{x}_{3}^{r}, x_{3}^{l}=\hat{x}_{3}^{l}$ and $\max \left\{x_{1}^{r}, x_{2}^{l}, x_{2}^{r}, x_{1}^{l}\right\}=\max \left\{\hat{x}_{1}^{r}, \hat{x}_{2}^{l}, \hat{x}_{2}^{r}, \hat{x}_{1}^{l}\right\}<\min \left\{\hat{x}_{3}^{r}, \hat{x}_{3}^{l}\right\}=\min \left\{x_{3}^{r}, x_{3}^{l}\right\}$. Also notice that equation (A.26) is always valid whether the standard is greater than $\bar{v}$ or not. So we have the same contradiction as in part (c) above.

## PROOF OF PROPOSITION 4:

Using Lemma A. 2 and Proposition A.1, we can easily get the following results:

$$
\begin{align*}
& \left(x_{2}^{r}\right)^{2}+\left(x_{2}^{l}\right)^{2}=\frac{4 \tau}{a}+\left(x_{2}^{r}-x_{1}^{l}\right)^{2}  \tag{A.30}\\
& \left(x_{1}^{r}\right)^{2}+\left(x_{1}^{l}\right)^{2}=\frac{4 \tau}{a}+\left(x_{1}^{r}-x_{2}^{l}\right)^{2} \tag{A.31}
\end{align*}
$$

So for solutions with quality standards lower than $\bar{v}$, we have six equations (A.13), (A.23)-(A.25), and (A.30)-(A.31), and six unknowns $x_{3}^{r}, x_{3}^{l}, x_{2}^{r}, x_{2}^{l}, x_{1}^{r}, x_{1}^{l}$. Let

$$
y_{i}^{j}=\sqrt{\frac{a}{4 \tau}} x_{i}^{j}, \quad c=\frac{B}{12 \sqrt{a \tau}} .
$$

Then we can get a system of equations concerning $y_{3}^{r}, y_{3}^{l}, y_{2}^{r}, y_{2}^{l}, y_{1}^{r}, y_{1}^{l}$ :

$$
\begin{aligned}
\left(y_{3}^{r}\right)^{2}+\left(y_{3}^{l}\right)^{2} & =1 \\
y_{2}^{r}+y_{3}^{l}-2 y_{3}^{r} & =c \\
y_{3}^{r}-y_{2}^{l}+y_{2}^{r}-y_{1}^{l} & =2 c \\
y_{2}^{l}+y_{1}^{r}-2 y_{1}^{l} & =3 c \\
\left(y_{2}^{r}\right)^{2}+\left(y_{2}^{l}\right)^{2} & =1+\left(y_{2}^{r}-y_{1}^{l}\right)^{2} \\
\left(y_{1}^{r}\right)^{2}+\left(y_{1}^{l}\right)^{2} & =1+\left(y_{1}^{r}-y_{2}^{l}\right)^{2}
\end{aligned}
$$

Unlike the majority voting case, some of the $y_{i}^{b}$ can be higher than one. There may be multiple solutions to the system of equations. The first possibility is the pro-minority power-switching equilibrium such that $y_{1}^{l}<y_{2}^{l}$. In particular, as $y_{1}^{l}$ goes to zero, both $y_{2}^{l}$ and $y_{1}^{r}$ go to one from the last two equations of the above system of equations. Then, $y_{2}^{l}+y_{1}^{r}-2 y_{1}^{l}=3 c$ implies that $c$ has to be smaller than $2 / 3$ to guarantee that such an equilibrium exists. For $c \leq 2 / 3$, we can solve the system of equations numerically. The second possibility is the pro-majority power-switching equilibrium such that $y_{1}^{l}>y_{2}^{l}$. In particular, as $y_{2}^{l}$ goes to zero, both $y_{1}^{l}$ and $y_{2}^{r}$ go to one from the last two equations of the above system of equations. Note that $c$ has to be larger than 10/29 to guarantee that such an equilibrium exists. For $c \geq 10 / 29$, we can also solve the system of equations numerically.

There can also be an equilibrium such that the equilibrium quality standards are $\bar{v}$. In particular, consider the glass-ceiling equilibrium where $\tilde{v}_{2}^{l}=v_{2}^{l}=\bar{v}$. Then the following inequality must be satisfied:

$$
\begin{equation*}
\frac{2}{3}\left[\frac{3}{2} \bar{v}+\frac{B}{4}+\frac{1}{2} \pi_{1}^{R}+\frac{1}{2} \pi_{2}^{R}\right] \leq \pi_{2}^{R}-\tau . \tag{A.32}
\end{equation*}
$$

By the fact that $x_{2}^{l}=0$ and equations (A.30)-(A.31), we can easily derive

$$
x_{2}^{r}=x_{1}^{l}=\sqrt{\frac{4 \tau}{a}} \quad \text { or } \quad y_{2}^{r}=y_{1}^{l}=1
$$

Therefore, the admission criterion in contentious states is exactly the same as the one in the majority voting case. Hence, we obtain the same expressions for values $\pi_{3}^{R}, \pi_{2}^{R}$, and $\pi_{1}^{R}$, and the admission criterion in homogeneous states is also exactly the same as the one in the majority voting case:

$$
\begin{aligned}
& y_{3}^{r}=\frac{1}{5}\left(\sqrt{4+2 c-c^{2}}-2 c+2\right) \\
& y_{3}^{l}=\frac{1}{5}\left(2 \sqrt{4+2 c-c^{2}}+c-1\right)
\end{aligned}
$$

Similar to the argument in step (ii) of the proof of Proposition 3, we need $c>10 / 29$ to guarantee inequality (A.32). Other types of equilibria may also exist under unanimity voting. For example, there may be an equilibrium such that in contentious states, only candidates of the minority type are admitted. But since the welfare of these equilibria cannot exceed that in the glass-ceiling equilibrium, we omit the discussion of these equilibria. (See the online Appendix for a complete equilibrium characterization.)

Finally, the long-term welfare under unanimity voting rule is given by

$$
U^{u}=3 E v+\frac{3}{2} a+\tau-2 \sqrt{a \tau} \gamma^{u}
$$

where $\gamma^{u} \equiv \frac{4 q_{3}}{y_{3}^{r}+y_{3}^{l}}+\frac{q_{2}}{y_{1}^{l}+y_{2}^{l}}\left(1+3\left(y_{1}^{l}\right)^{2}+3\left(y_{2}^{l}\right)^{2}\right)$. Comparing the welfare of different equilibria gives us the proposition. In particular, we numerically find that the pro-minority power-switching equilibrium dominates the pro-majority power-switching equilibrium in efficiency when $c<0.47$, and vice versa; the pro-majority power-switching equilibrium dominates the glass-ceiling equilibrium in efficiency when $c<1.97$, and vice versa.

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    ${ }^{\dagger}$ Go to https://doi.org/10.1257/mic. 20160237 to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.
    ${ }^{1}$ See March (1962) and Pfeffer (1981) for discussions of internal politics. Other related discussions can be found in Milgrom and Roberts $(1988,1990)$ and Meyer, Milgrom, and Roberts (1992).

[^1]:    ${ }^{2}$ Models with other voting rules are considered in Barberà, Maschler, and Shalev (2001) and Granot, Maschler, and Shalev (2003).
    ${ }^{3}$ Acemoglu, Egorov, and Sonin (2010) also present a normative investigation of how the degree of incumbency veto power affects the quality of government, which is a very different question from ours.

[^2]:    ${ }^{4}$ The collective search model with heterogeneous committee member types and multidimensional candidates is in general very complicated to solve analytically. For example, Compte and Jehiel (2010) focuses on the limiting case where the discount rate goes to one. In the online Appendix, we numerically solve the five-member model and show that the qualitative results of the baseline model still hold.
    ${ }^{5}$ In our model random exits from the club serve the role of discounting, thus no discounting over time is needed. In the online Appendix, we extend the model to allow for more general discounting.
    ${ }^{6}$ Depending on the applications, type can be interpreted as race, gender, ideology (or party affiliation), or specialization. The exogenous type assumption is plausible because these types are either fixed (e.g., race and gender) or very difficult to change (e.g., ideology and specialization).
    ${ }^{7}$ For example, one can imagine that the club elects a chairman by majority voting, who then decides on distributing some monetary or nonmonetary resources (e.g., research funds, office spaces, other perks). The elected official is loyal to his "party," and distributes the rent to members of his type only.

[^3]:    ${ }^{8}$ As shown by Acemoglu, Egorov, and Sonin (2015), one of the crucial sufficient conditions to guarantee equilibrium existence in the club formation literature is a "single crossing" assumption, which in our context would imply that right-type members would have higher stage payoffs in the more "right" states. This does not hold in our model, as right-type incumbents prefer the state with two right-type incumbents to the state with three right-type.
    ${ }^{9}$ It is always desirable for the candidate to join the club because, as shown later, the club members always receive positive payoffs when the outside option is normalized to zero.
    ${ }^{10}$ Such a cost can take many forms, e.g., reviewing files, interviewing, meetings, and the opportunity costs of leaving the position vacant.

[^4]:    ${ }^{11}$ Condition (3) is implicitly built upon the one-shot deviation principle. In our model, a one-shot deviation is a deviation just in a single round of a particular period, while a general deviation may involve several deviations in many rounds of many periods. In the online Appendix, we show that the one-shot deviation principle still applies in our context.
    ${ }^{12}$ Ruling out equilibria of coordination failure in voting is common in the literature, i.e., voting "no" on a preferred outcome is a weakly dominated best response if everyone else does so (see, e.g., Chan et al. 2018). A "trembling hand" argument ensures that voters do not use weakly dominated strategies because there is always a positive probability to be pivotal. Alternatively, if incumbent members vote sequentially in each selection round, then they will vote their true preferences as well.
    ${ }^{13}$ It is not a Markov equilibrium to set $v_{i}^{b^{\prime}}=\bar{v}$ and not admit any candidate because the equilibrium search payoff would be $\pi_{i}^{R}=-\infty$, and it is easy to see that this strategy is dominated.

[^5]:    ${ }^{14}$ One imagines that the founders of the club would want to ensure that the club selects the most efficient equilibrium and commits to the optimal rule that achieves this equilibrium. For example, Barzel and Sass (1990) provides evidence that developers of condominiums choose voting rules for condominium homeowner's associations.

[^6]:    ${ }^{15}$ This is analogous to option value increasing in the variance of the return of the underlying asset.

[^7]:    ${ }^{16}$ This is because a new member of quality $v$ contributes a value of $v$ in each period he remains in the club, and he is in the club for sure in the period he is admitted, and has a survival chance of $2 / 3$ in each of the future periods.
    ${ }^{17}$ The reason $w \geq \underline{v}$ is that the club can always admit everybody (i.e., $\hat{v}=\underline{v}$ ).

[^8]:    ${ }^{18}$ As shown by Cai and Feng (2007), an early version of this paper, the result holds for an arbitrary number of members and any distribution of quality.

[^9]:    ${ }^{19}$ The existence of the glass-ceiling equilibrium crucially depends on the assumption that $v$ has bounded support: when the support is unbounded, there is no glass ceiling because it is suboptimal to reject a candidate whose quality is extremely high. In the online Appendix, we study the case where $v$ follows an exponential distribution and show that our main results (Proposition 5) still hold.

[^10]:    ${ }^{20}$ To simplify comparison, we normalize admission probabilities by multiplying them by $\sqrt{a / \tau} / 2>1$. The normalized admission probabilities take values between zero and one.

[^11]:    ${ }^{21}$ For some range of $c$, there exist other types of equilibria that are easily dominated. Proposition A. 1 in the online Appendix provides a complete equilibrium characterization under unanimity voting.

[^12]:    ${ }^{22}$ In the online Appendix, we characterize all different kinds of equilibria under unanimity voting. For $c>1.97$, there still exists a power-switching equilibrium under unanimity voting, in which incumbents of the two types engage in intensive politicking and create very long delays in the admission of a new member. It is shown that such an equilibrium is worse than the glass-ceiling equilibrium under majority voting.
    ${ }^{23}$ It would be an interesting empirical question to distinguish preference aggregation from information aggregation in collective decisions, and to test the different predictions from the two theoretical perspectives.
    ${ }^{24}$ Such as "A committee is a thing which takes a week to do what one good man can do in an hour" by Elbert Hubbard.

[^13]:    ${ }^{25}$ Albrecht, Anderson, and Vroman (2010) also show that unanimity voting is optimal in a collective search model, but in a limiting sense when search cost per round goes to zero (in their model the discounting factor between search rounds goes to one).
    ${ }^{26}$ For another example, due to cultural influences, firms in Japan, Korea, and other East Asian countries tend to emphasize consensus building as a distinct trait of corporate culture. In some Japanese companies, agreement is normally obtained by circulating a document that must first be signed by the lowest level manager, and then upward, and may need to be revised, and the process may have to start over again (Verma 2009). As argued by Ouchi (1981), the consensus-based decision-making style in Japan improves firm performance, which is supported by several empirical studies (see, e.g., Dess 1987). However, other factors not considered here can be important in optimal corporate decision-making structures. For instance, consensus decision-making tends to be more conservative and less likely to take risks, which may or may not be an advantage depending on the environment.
    ${ }^{27}$ In an interesting study, Milliken and Martins (1996) finds that diversity has negative effects on group outcomes early in a group's life, but after this stage, once a certain level of behavioral integration has been achieved, groups may be able to obtain benefits from diversity.
    ${ }^{28}$ Consider, for example, an academic department that is recruiting new faculty members. Our analysis suggests that in order to achieve the right degree of incongruity, the department can change the minimum requirement for a candidate to qualify for consideration (corresponding to $a$ ), the number of profiles that must be read by the recruiting members (corresponding to $\tau$ ), and the amount of discretionary resources (corresponding to $B$ ).

