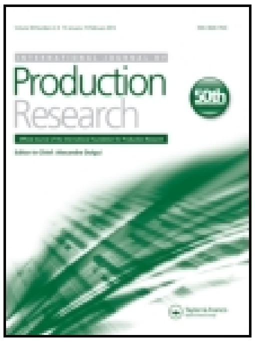

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Demand information sharing and channel choice in a dual-channel supply chain with multiple retailers

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In this paper, under a dual-channel supply chain consisting of a manufacturer and multiple retailers, we investigate vertical and horizontal information sharing in different channel structures and the manufacturer's choice on whether or not to keep a direct channel. To this end, we first study the dual-channel structure where uncertain demand is a linear function of price with a generalised-distribution base demand and show that the retailers have incentives to share information horizontally but not vertically, while the manufacturer is better off with vertical information sharing but its expected profit is not affected by horizontal information sharing. We next examine the retail-channel structure and find the basic results remain unchanged. Finally, we provide closed-form internal and external conditions under which the manufacturer can benefit from owning a dual-channel structure. Our study extends the existing literature by combining information sharing and dual-channel choice, introducing channel difference, discussing the impact of channel structure on horizontal and vertical sharing as well as providing interesting managerial insights for channel choice.

Keywords: information sharing; dual-channel; channel choice; demand uncertainty; game theory; supply chain

1. Introduction

In the last couple of decades, we have witnessed the rapid development of E-commerce and Third Party Logistics, which provides an opportunity for the manufacturer to market directly to its customers. Besides the traditional intermediaries channel, the dual-channel supply chain also includes direct marketing. The most common direct channel is online trading, using the convenience of Internet technology. Setting up a direct online store can bring advantages such as better demand visibility, closer customer contract and higher profit margins. Many famous brands own dual channels, such as Hewlett-Packard, Eastman Kodak and Apple.

However, how to efficiently utilise the direct channel is always a challenge for many manufacturers. For example, Levi Strauss & Co. discontinued its direct Internet channel and handed online sales over to the e-retail partners. Among the top 10 US and Canada Internet retail companies in 2012, only two (Dell.com and Apple.com) were manufacturer's direct channels, whereas five were owned by physical store retailers (Staples, Wal-Mart, Office Depot, Liberty Interactive and Sears), and the rest were pure-play e-retails (Amazon.com, Netflix.com and CDW.com) (Top 500 guide Website 2013). One question is raised here: For a manufacturer owning dual-channel structure, under what conditions should the manufacturer keep or close the direct channel to maximise profit? To provide a clear analysis of the problem, we first discuss the issue of information sharing.

One of the most important reasons for a manufacturer to consider a dual-channel structure is market demand uncertainty. Having a direct channel can help the manufacturer observe market information and lessen the impact of demand fluctuation. Therefore, demand information plays an important role in channel structure selection. Retailers, downstream of the manufacturer, own private information and can choose to share or not share this information. The retailers' decision on information sharing greatly impacts the manufacturer's channel structure decision. If retailers are willing to share information with the manufacturer, the function of using a direct channel to reduce market uncertainty will become weak. Once a direct channel is set up, the manufacturer will have the opportunity to meet the market demand directly, so the information structure in the supply chain will change. In this situation, retailers will rethink the optimal strategy of information sharing. If private information is shared with the upstream companies to reduce the wholesale price, sharing information could possibly be a better decision than not sharing information. Therefore, channel structure also

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makes a difference in the retailers' decision on information sharing. As such, the retailers' information sharing and manufacturer's channel choice greatly influence one another. Based on these observations, our objective is to analyse the retailers' sharing behaviour in two different channel structures and examine its influence on channel choice. One of the motivations for our paper is to incorporate the influence of information sharing so that research on channel choice will be more meaningful.

Information sharing can help the upstream manufacturer respond to market demand quicker and reduce the influence of demand variability. Previous researches such as Vives (1984), Li (1985), Shapiro (1986), Raith (1996), Lee, So, and Tang (2000), Raghunathan (2001), Li (2002) and Li and Zhang (2008) have discussed the issue in a supply chain with a traditional retail channel or in an oligopoly market. Most of them focus on the value of information sharing to manufacturing, inventory management, ordering, contract and coordination. What is not clear is the impact of information sharing on pricing and channel choice in a dual-channel supply chain with one-manufacturer and multi-retailers. Compared with a single retail channel, what are the equilibrium outcomes of horizontal and vertical information sharing under a dual-channel structure? With these questions as our research background, our paper attempts to study information sharing and channel choice from the manufacturer's view of maximising expected profits.

Many related literature examines the impact of setting up direct channel on traditional retail channels (Balasubramanian 1998; Hendershott and Zhang 2006; Arya, Mittendorf, and Sappingtou 2007). There are also researches such as Park and Keh (2003), Chiang, Chhajed, and Hess (2003), Liu and Zhang (2006), Dumrongsiri et al. (2008) and Cai (2010) investigating whether a direct channel should be set up or not. However, there is little research combining the issues of information sharing and channel choice in a dual-channel supply chain with multiple downstream retailers. The detailed conditions for keeping or closing the dual-channel structure are not well presented. In our paper, we will explore the following questions:

(1) In two channel structures, will the downstream retailers share demand forecast with the manufacturer and the other retailers? What are the impacts on the manufacturer's profit?
(2) What is the trade-off for channel choice based on the analysis of information sharing?
(3) Under what situations will a manufacturer gain more profit after opening the direct channel?
(4) How do demand uncertainty, forecast accuracy and price sensitivities in two channels affect the decision variables?

Our paper has several contributions. Vertically, retailers have no incentive to share private information with the upstream companies, regardless of the channel structure. This finding is consistent with the existing literature which mostly considers a single-channel structure. The reason is that non-participating retailers can infer demand information of the manufacturer from the wholesale price, and participating retailers do not obtain additional benefits from information sharing. This is the negative outcome of the 'leakage effect'.

In a dual-channel structure, vertical information sharing can cause an increase in the direct price and wholesale price, which tends to reduce direct demand. The overall effect for the retailers' profits is negative. Therefore, not disclosing information is the only Bayesian Nash equilibrium. By comparing the outcomes in different channel structures, we do not find any differences in outcome. As such in a market environment, the manufacturer's channel decision can be made without the consideration of information sharing, because retailers will not share information unless some contracts are signed to earn information rent. This kind of conclusion is well supported by the literature, and also a foundation of the incentive mechanism research.

Horizontally, a non-participating retailer would like to join the information alliance in order to gain additional profit, and no participating retailer would want to quit the alliance. As such, the retailer's best response is to participate in horizontal information sharing. Information sharing is always beneficial for the participating retailers. The reason is that sharing information can lead to an increase in their share of the retail market. The results are consistent with prior research in an oligopoly market (Li 1985; Gal-Or 1986; Raith 1996). For the manufacturer, the retailers' private information is always valuable and can be used to predict more accurate demand. However, the horizontal information sharing occurs between retailers only. In a market with uncertain demand, the manufacturer's expected profit is not affected since it cannot obtain the retailers' information. Therefore, the manufacturer's expected profit is not affected, irrespective of whether the retailers share information among each other.

In addition, when a new direct channel is added, the above results remain unchanged. Therefore, in a free market, the manufacturer can choose a channel structure without worrying about the retailers' decisions on information sharing. This conclusion is very useful when a manufacturer is concerned that a new channel can change the downstream information sharing situation. We also find that different price sensitivities in the two channels greatly impact the channel choice. Low price sensitivity in the direct channel and high price sensitivity in the retail channel provide a favourable market environment for the dual-channel structure. Demand uncertainty encourages the manufacturer to open a new direct channel.

In the following sections, we first review the related literature on information sharing and channel choice and point out our contribution. Then we present the model and assumptions and study the issue of vertical forecast sharing decision. Next, the paper discusses whether the retailers should share demand forecast horizontally and what factors will contribute to increased firm profits. Based on the analysis of information sharing, we continue to explore the manufacturer's channel choice issue. Finally, we provide managerial sights and concluding remarks.

2. Literature review

The study of dual-channel distribution systems based on B2C E-commerce has become increasingly more important in supply chain management. The key research issues involve multi-channel structure arrangement, pricing, information sharing, channel conflict and coordination, and channel operation which includes manufacturing, inventory and ordering strategies. We focus on two issues in the related literature: (1) channel structure choice and (2) information sharing.

2.1 *Channel structure choice*

In the E-commerce, the issues of pricing strategy, channel coordination and channel choice in a dual-channel structure have recently received greater attention. Initial research in the area focuses on the impact of setting up a direct channel on traditional retail channels. Hendershott and Zhang (2006) examine a model in which an upstream firm can sell directly online and through heterogeneous intermediaries to heterogeneous consumers. They find in the dual-channel structure that competition and segmentation result in lower intermediary prices. Arya, Mittendorf, and Sappingtou (2007) conduct similar research and they show that the retailer can benefit from the manufacturer's encroachment by opening a sales channel. Wu, Petruzzi, and Chhajed (2007) examine two manufacturers selling a substitutable product through a decentralised or integrated retailer. Xia and Zhang (2010) find that adding an Internet channel can enable a company to expand their markets. Cao, Jiang, and Zhou (2010) discover that demand uncertainty plays an important role in affecting the competing manufacturers' decisions on channel structure.

Another research area is whether a direct channel should be set up. Park and Keh (2003) build a price-competing model without considering market demand uncertainty. Their results show that the manufacturer benefits from the dual-channel structure, but the arrangement is not as beneficial to the retailer from a profit perspective. Chiang, Chhajed, and Hess (2003) examine the degree to which customers accept a direct channel as a substitute for retail channel. They show how direct marketing can indirectly increase the manufacturer's profit. Similar studies carried out are as follows: Yao and Liu (2005) hypothesise that the demand is not only affected by price but also by the retailer's value-added activities; Liu and Zhang (2006) explore channel interactions in an information-intensive environment; Yue and Liu (2006) discuss the influence of direct channel in a single retailer supply chain with uncertain demand; Yoo and Lee (2011) find that the impact of the Internet channel varies substantially across channel structures and market environment; and Cai (2010) investigates the influence of channel structures and channel coordination in the context of two single-channel and two dual-channel supply chains. In a two-period supply chain, Xiong et al. (2012) find under certain conditions the retailer and the entire supply chain benefits when the manufacturer sets up an online sale channel.

2.2 *Information sharing*

More recent research examines the behaviour of information sharing in the traditional retail channel, including vertical sharing between manufacturer and retailers and horizontal sharing between the retailers. There are several studies on horizontal information sharing with firms operating in an oligopoly market such as Clarke (1983), Vives (1984), Gal-Or (1986), Li (1985), Shapiro (1986) and Raith (1996). These classic papers find that the equilibrium outcome involves no sharing of sales demand information but price information is disclosed. Shi et al. (2013) study horizontal information sharing between two suppliers (one is considered a strategic supplier and the other, a backup supplier) who provide components for a manufacturer. They find sharing information or cooperation between two suppliers is beneficial for the suppliers but not the manufacturer. Our paper extends prior research by exploring these issues in a two-echelon supply chain with a dual-channel structure.

As for vertical information sharing, the literature mainly discusses the value of information on manufacturing, inventory management, ordering, contract and coordination. Lee, So, and Tang (2000) find that with uncertain demand, the manufacturer's profit will increase due to sharing information among the retailers. Cachon and Lariviere (2001) prescribe a mechanism for credible sharing of demand information. Li (2002) examines the leakage effect brought by the information sharing in a supply chain. Chen (2003) focuses on modelling of incentive mechanisms designed to optimise the profit of the supply chain. Li (2002) and Li and Zhang (2008) study the confidentiality issue of information exchange. They find that all parties would like to share information if the retail competition is intense and confidentiality is maintained. Li and Zhang (2008), Yao, Yue, and Liu (2008), Babich et al. (2012) and Datta and Christopher (2011) also contribute greatly to the study of vertical information sharing. Datta and Christopher (2011) show that widespread distribution, rather than a centralised approach, can effectively improve the management of supply chains. Their results provide good support for the influence of retailers' information sharing which can mitigate the effects of demand uncertainty.

Research on dual-channel structure and information sharing in a supply chain is limited. Compared with the single retail channel structure, dual-channel involves a manufacturer and multiple retailers both cooperating and competing with each other in the original retail channel; but in the new direct channel, they are faced with potential channel conflict. Therefore, the problem of information sharing becomes much more complex in a dual-channel structure.

In a dual-channel supply chain with demand uncertainty, Huang, Yan, and Guo (2007) demonstrate the existence of the bullwhip effect and the essential value of information sharing. Yue and Liu (2006) assume that the Gaussian demand is a linear function of price, and the manufacturer and retailer both have private demand information. They compare the firms' equilibrium profits with and without sharing demand forecast under the two scenarios of MTO (make-to-order) and MTS (make-to-stock) products. They find that under certain conditions, the retailer is better off with vertical information sharing, while the manufacturer always benefits. In Mukhopadhyay, Yao, and Yue (2006), the retailer decides what value to add to the product and the price of the augmented product. Ruiliang and Sanjoy (2010) investigate the value of forecast information based on the Bertrand game and find that the online and traditional retailers always benefit from high forecast accuracy. Yan and Pei (2011) build a bargaining model to coordinate profit sharing and find that the retailer is not affected by sharing their private market demand information.

Our research is different from the literature as follows:

(1) We study the information sharing strategy of multiple retailers vertically and horizontally in a dual-channel structure. Previous research only examines the single-channel structure and ignores the influence of adding a direct channel when retailers' information sharing is present. We study this combination and observed some interesting results.
(2) Information on uncertain demand matters a lot in channel choice of the manufacturer. Based on the analysis of information sharing, we discuss the retailers' strategy in different channel structures and help the manufacturer to position itself to address the market situation.
(3) The extant literature assumes the cross-price sensitivities are the same in two channels, while our paper relaxes the assumption. We discuss the influence of price sensitivity on channel choice.

3. The model

Our business scenario is presented in Figure 1. Here, a simple supply chain is made up of n retailers ($n \geq 1$) and one manufacturer who already owns a long-established traditional channel by cooperating with the retailers. At the same time, the manufacturer can decide whether to set up a direct channel or not, and the retailers cannot stop the behaviour of direct trading, but could choose whether to share the private information about market demand and set the product sales price.

We make several key assumptions which are consistent with previous research (Ingene and Parry 2000; Park and Keh 2003; Li and Zhang 2008). Fixed cost is not considered since it does not affect the decision variables. In addition, retailers are price takers to the wholesale price and the manufacturer's price to consumers is higher than manufacturer's price to retailers. We assume the direct price to customers is higher than the wholesale price to the retailers. In addition, in our model, the manufacturer provides a common base product to the retailers. Each retailer further customises the base products to make them similar but not identical (Li and Zhang 2008).

To facilitate understanding of the model, we first provide the symbols used and their corresponding meanings in Table 1.

Based on the above assumptions, we derive the following demand functions:

$$q_0 = a_1 - b_1' p_0 + d \sum_{j=1}^{n} (p_j - p_0) \tag{1a}$$

$$q_i = a_2 - b_2' p_i + d \sum_{j=0\, J \neq i}^{n} (p_j - p_i) \tag{1b}$$

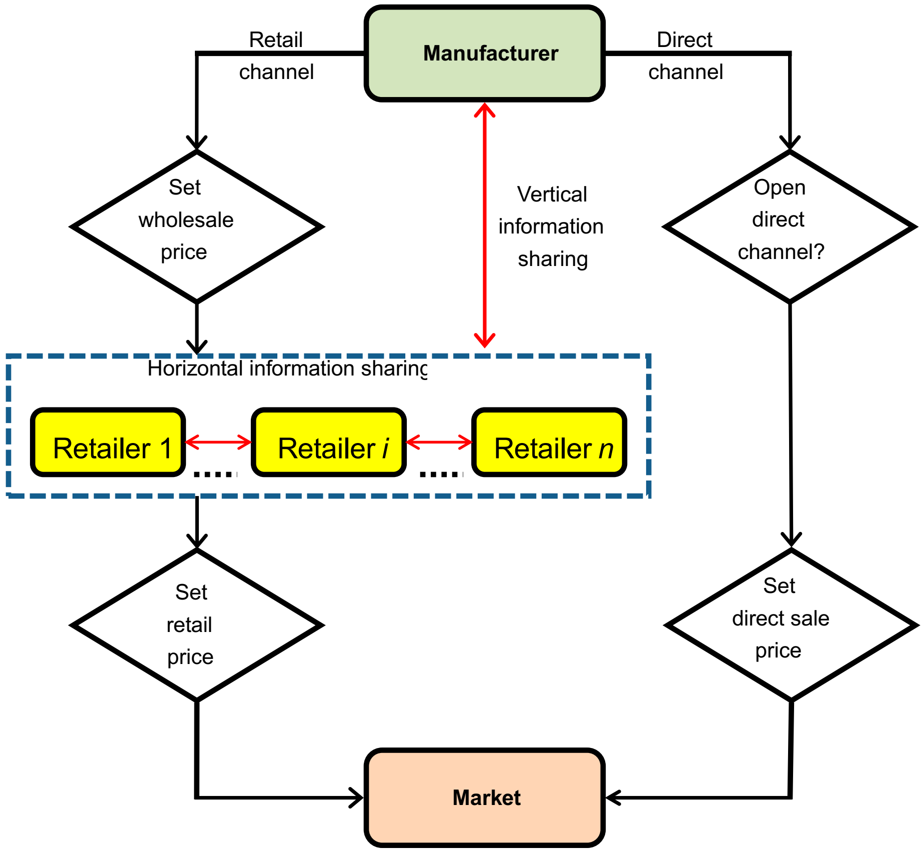


Figure 1. Dual-channel structure.

Table 1. Definition of symbols.

Symbol	Definition
n	Number of retailers
q_0	The manufacturer's direct channel demand
q_i	Retailer i's demand
D_r	The retail channel total demand
λ	Degree of consumer preference for direct channel over the retail channel
w	Wholesale price
p_0	Direct channel price
p_{i}	The retailer i's price
θ	A variable describing the uncertainty of market demand
Y_0	The private information about θ owned by the manufacturer
Y_i	The private information about θ owned by the retailer i
c_d	Marginal direct channel cost for the manufacturer
c_r	Marginal retail cost for the retailer
a	Base market demand in single-channel scenario
a_1	Direct channel base demand in dual-channel scenario
a_2	Retail Channel base demand in dual-channel scenario
b	Retailer i's own price sensitivity in single-channel scenario
b_1	Direct channel's own price sensitivity in dual-channel scenario
b_2	Retail channel's own price sensitivity in dual-channel scenario
d	Cross-price sensitivity between the firms

Equation (1a) is the direct channel demand, which is a linear function of prices in the two channels. a_1 is the original base demand in direct channel. The direct demand of the manufacturer is negatively related with its price in the direct channel and positively related with the difference its price is above the competitor's price. Similarly, Equation (1b) is the demand for retailer i. a_2 is the original base demand for every retailer. The demand of a retailer is negatively related with its price and positively related with the difference its price is above the competitor's price. Here, b_1' is the direct channel's pure price sensitivity after considering cross-price sensitivity, and $b_1' + dn$ represents the direct channel's own price sensitivity, and $b_2' + dn$, d and b_2' for the retailer i have similar meanings. Let $b_1 = b_1' + dn$ and $b_2 = b_2' + dn$. Thus, we rewrite the above demand functions and get Equations (2a) and (2b). We can see that b_1 and b_2 is greater than or equal to dn. This means that 'own price sensitivity' is greater than or equal to 'cross-price sensitivity'.

$$q_0 = a_1 - b_1 p_0 + d \sum_{j=1}^{n} p_j \tag{2a}$$

$$q_i = a_2 - b_2 p_i + d \sum_{j=0, j\neq i}^{n} p_j \tag{2b}$$

If no direct channel is opened, the market demand function for retailer i can be simplified as $q_i = a - bp_i + d\sum_{j=1,\, j\neq i} p_j$. This type of demand function is widely used in the literature (Tsay and Agrawal 2000; Yao and Liu 2005; Mukhopadhyay, Yao, and Yue 2006). In addition, the demand function in the direct channel is linear (Chiang, Chhajed, and Hess 2003).

To capture uncertainty in market demand, we assume that θ is a random variable to describe demand volatility. Here, we have $E[\theta] = 0$, $\mathrm{Var}[\theta] = \sigma^2$. Then the channel demands can be rewritten as follows:

(a) Single retail channel:

$$q_i = a + \theta - bp_i + d \sum_{j=1, j\neq i}^{n} p_j, \quad \text{here}\, b > d(n-1)$$

(b) Dual channel:

$$q_0(p) = a_1 + \theta - b_1 p_0 + d \sum_{j=1}^{n} p_j$$

$$q_i(p) = a_2 + \theta - b_2 p_0 + d \sum_{j=0, j\neq i}^{n} p_j \quad j = 0, 1, \ldots n \quad i = 1, 2, \ldots, n$$

Unlike previous research, we believe that the manufacturer has the ability to observe the market, especially in the dual-channel structure. As such, the manufacturer has information Y_0 on market demand. As a result of direct customer contact, the retailers have their own private information $Y_i (i \in I)$. We provide assumptions from Li (2002) on the information structure below:

Assumption 1: $E[Y_i|\theta] = \theta$, $E[Y_0|\theta] = \theta$. That is, Y_0 and Y_i are unbiased estimators of θ.

Assumption 2: $E[\theta|Y_1, \ldots, Y_n, Y_0] = \beta + \sum_{i\in N} \beta_i Y_i + \beta_d Y_0$, where β, β_i and β_d are constants.

Assumption 3: Y_i and Y_0 are identically distributed on θ.

Therefore, the firms' private demand signals have a symmetric probability distribution. We use the following variables from Li (2002): $s_0 = E[Var[Y_0|\theta]]/Var[\theta]$ and $s = E[Var[Y_r|\theta]]/Var[\theta]$ to capture information accuracy of the manufacturer and retailer, respectively. The reciprocal, $1/s$, is an indicator of signal accuracy. Obviously, the greater the value of s, the less amount of information is available.

Assume that $\mathrm{K} \subseteq \mathrm{N}$ is the set of retailers who share private information about market demand, where $\|\mathrm{K}\| k, k = (0, 1, \ldots, n)$ is the number of retailers involved in information sharing. We define $\mathrm{K}_0 = \mathrm{K} \cup \mathrm{Y}_0$ as the set including the manufacturer and retailers who share information. Let $Y_k = \{Y_j\}_{j\in K}$ and $Y_{k_0} = \{Y_j\}_{j\in K_0}$ be the set of disclosed demand signals, including the manufacturer's demand signal.

Lemma 1: $\forall K \subseteq N, \forall i \in N \backslash K$, we can get:

$$E[Y_i \big| \{Y_j\}, j \in K_0] = E[\theta \big| \{Y_j\}, j \in K_0] = \frac{t}{k+s+t} Y_0 + \frac{1}{k+s+t} \sum_{j \in K} Y_j$$

Particularly, when the information set is owned by the firms is Y_0 or $\{Y_j, Y_0\}$, we have $E[Y_i|Y_0] = E[\theta|Y_0] = \frac{t}{s+t} Y_0$, $E[Y_i \big| Y_j, Y_0] = E[\theta \big| Y_j, Y_0] = \frac{t}{1+s+t} Y_0 + \frac{1}{1+s+t} Y_j$, here $t = s/s_0$. (see Appendix 1).

According to Lemma 1, we find that given Y_{K_0}, Y_0 and $\sum_{j \in K} Y_j$ are sufficient to predict θ and other demand signals. This kind of prediction is a linear combination of known demand signals with forecast accuracy as coefficients. It is obvious that information accuracy has an important influence on estimating information value. We have two extreme scenarios: (1) when $s_0 = 0$, t approaches infinity and the manufacturer has complete information about market demand; (2) when $s_0 = +\infty$, t becomes 0 and the manufacturer has no information about market demand.

We first analyse the issue of information sharing under demand uncertainty by examining both the dual-channel and single-channel structure, and then quantifying the effects of demand forecast sharing on profits. Next, we study the decision of channel choice.

4. Vertical information sharing

With a vertical information sharing arrangement, a participating retailer transmits its information to a depository that is accessible to the manufacturer and all participating retailers. Non-participating retailers can infer the shared information from the manufacturer's wholesale price decision.

We use game theory to examine the decision sequences as follows:

Stage One	Each retailer commits to either share its information or not. After that, retailer i observes a signal Y_i about the demand forecast
Stage Two	The manufacturer sets a wholesale price w and the direct sale price
Stage Three	After obtaining the wholesale price w, each retailer decides on a retail price p_i

The manufacturer has access to the information set $Y_{K_0}(Y_0, Y_j, j \in K)$ to make decisions on the wholesale price w, which is a function of Y_{K_0}. We restrict the search for equilibrium to the subspace where w is related to Y_{K_0} only through a monotonic relationship with $\mathrm{E}[\theta|\mathrm{Y}_{\mathrm{K}_0}]$. According to the equilibrium outcome computing results on $\mathrm{E}[\theta|\mathrm{Y}_{\mathrm{K}_0}]$, we find that w is a strictly increasing function of $\sum_{j \in K} Y_j + t\ Y_0$.

Next, we examine the situation where the retailers do not share information. Although the retailers cannot observe Y_{K_0} directly, they will try to infer it from w. It is reasonable to assume the retailers know the monotonic relationship between Y_{K_0} and $\mathrm{E}[\theta|\mathrm{Y}_{\mathrm{K}_0}]$. Thus, the non-participating retailers can infer the sum of total demand $\sum_{j \in K} Y_j + t\ Y_0$, which is called 'information leakage effect'. In the next few sections, we derive the outcomes of each stage, working backwards from stage three to stage one.

4.1 *Stage three outcome: retailers' price decision*

In the last stage, given the vertical information sharing arrangement ($K \subseteq N$, $k = 0, 1, \ldots, n$), the wholesale price and the direct sale price, the retailers need to set retail price to maximise the conditional expected profit:

$$E[\pi_i|Y_i, w] = \left\{ a_2 + E[\theta|Y_i, w] - b_2 p_i + d p_0 + d \sum_{j \neq i, j=1}^{n} E[p_j|Y_i, w] \right\} (p_i - w)$$

According to the first-order condition, we have

$$2 b_2 p_i^* = a_2 + E[\theta|Y_i, w] + b_2 w + d p_0 + d \sum_{j \neq i, j=1}^{n} E[p_j^*|Y_i, w]$$

The equilibrium retail price every retailer sets is a monotonic function of the information it is able to access. Given any information sharing arrangement ($K \subseteq N$, $k = 0, 1, \ldots n$), we can only have one Bayesian Nash equilibrium (see Appendix 2) in stage three:

$$i \in K, \quad p_i^*(Y_{K_0}) = \frac{1}{2b_2 - dn + d}\left(a_2 + b_2 w + dp_0 + \xi_1 \sum_{j \in Y_K} Y_j + \xi_2 Y_0\right)$$

$$i \notin K, \quad p_i^*(Y_i, w) = \frac{1}{2b_2 - dn + d}\left(a_2 + b_2 w + dp_0 + \eta_1\left(\sum_{j \in Y_K} Y_j + tY_0\right) + \eta_2 Y_i\right)$$

Here we have $\xi_1 = \frac{1}{k+s+t}, \xi_2 = t\xi_1; \eta_1 = (1-\eta_2)\frac{1}{k+s+t}, \eta_2 = \frac{2b_2 - dn + d}{2b_2(k+s+t+1) - d(n-k-1)}$ to simplify our formulas.

According to the above equilibrium, the retail price is an increasing function of $E[\theta|Y_{K_0}]$ (or $\sum_{j \in K} Y_j + t\, Y_0$). With a more favourable market indicating greater demand, the retail price will be higher. η_2 is a decreasing function of k. For non-participating retailers, the conditional expectation of their retail price is the same as the equilibrium price of participating retailers.

The retailer's order quantity is: $E[q^*_{i \in N \setminus K}|Y_i, w] = E[q^*_{i \in K}|Y_{K_0}] = b_2[p_i^*(Y_{K_0}) - w]$

The total demand in the retail channel is: $E[D_{M\_d}] = E[\sum_{i \in N} q^*_{i \in K}(Y_i, w)] = nb_2[p_i^*(Y_{K_0}) - w]$

4.2 *Stage 2 outcome: manufacturer's price decision*

Given the retailers' equilibrium prices, the manufacturer sets the wholesale price and the direct sale price to maximise the conditional expected profit:

$$E[\pi_{M\_d}|Y_{K_0}] = E\left[w \sum_{j \in N} q_j^*(Y_j, w) + (p_0 - c_d)(a_1 + \theta - b_1 p_0 + d\sum_{j \in N} p_j^*)|Y_{K_0}\right]$$

The direct price should be higher than the wholesale price to avoid the retail channel from purchasing products in the direct channel, which is an arbitrage constraint (Chiang, Chhajed, and Hess 2003 and Xiao, Choi, and Cheng 2013). We discuss two situations for the above optimisation problem subject to $w \le p_0$, below.

(1) $w < p_0$, which needs $c_d > \frac{a_2 b_1 - a_1 b_2 + (a_1 - a_2)dn + (b_1 + b_2)E[\theta|Y_{K_0}]}{d(b_1 - dn) + b_1(b_2 - dn)}$ to be true;

(2) Specially, $w = p_0$, which needs $c_d < \frac{a_2 b_1 - a_1 b_2 + (a_1 - a_2)dn + (b_1 + b_2)E[\theta|Y_{K_0}]}{d(b_1 - dn) + b_1(b_2 - dn)}$ to be true.

For the first situation, given the information arrangement ($K \subseteq N$, $k = 0, 1, \ldots n$), the manufacturer's equilibrium prices are:

$$p_0^* = \frac{(b_2 + d)(a_1 + b_1 c_d + E[\theta|Y_{K_0}]) - dn(a_1 - a_2 + c_d(b_1 + d))}{2b_1(b_2 + d) - 2dn(b_1 + d)}$$

$$w^* = \frac{a_2 b_1 + a_1 d + (b_1 + d)E[\theta|Y_{K_0}]}{2b_1(b_2 + d) - 2dn(b_1 + d)}$$

We note that w is the monotonic increasing function of $E[\theta|Y_{K_0}]$. For the second situation,
$w = p_0 = \frac{a_2 b_2 n + a_2 dn - b_2 c_d dn - c_d d^2 n + (2b_2 + d + b_2 n)E[\theta|Y_{K_0}] + (a_1 + b_1 c_d)(2b_2 + d - dn)}{2(b_1(2b_2 + d - dn) - n(-b_2^2 + d^2 + b_2 d(1+n)))}$.

4.3 *Stage one outcome: decision on vertical information sharing*

Given the pricing strategies in the last two stages, we can compute every firm's expected profit.

For all the retailers, the expected profit function can be expressed as:

$$E[E[\pi_{R\_d}|Y_i, w]] = b_2 E[(p_i^* - w)^2]$$

We define the expected profit of participating and non-participating retailers, respectively, as $\pi^{S*}_{R\_d}(k)$ and $\pi^{N*}_{R\_d}(k)$. The expected profit of the manufacturer is $\pi^{*}_{M\_d}$.

When $c_d > \frac{a_2b_1 - a_1b_2 + (a_1 - a_2)dn + (b_1 + b_2)E[\theta|Y_{K_0}]}{d(b_1 - dn) + b_1(b_2 - dn)}$, this means $w < p_0$,

$$\partial_k \pi^*_{M\_d} = \frac{(2b_2^2 + 3b_2d + d^2 + b_2(b_1 + d)n)s\sigma^2}{4(2b_2 + d - dn)(b_1(b_2 + d) - d(b_1 + d)n)(1 + k + s + t)^2}$$

When $c_d < \frac{a_2b_1 - a_1b_2 + (a_1 - a_2)dn + (b_1 + b_2)E[\theta|Y_{K_0}]}{d(b_1 - dn) + b_1(b_2 - dn)}$, this means $w = p_0$,

$$\partial_k \pi^*_{M\_d} = \frac{(2b_2 + d + b_2n)^2 s\sigma^2}{4(2b_2 + d - dn)(b_1(2b_2 + d - dn) - n(-b_2^2 + d^2 + b_2d(1 + n)))(1 + k + s + t)^2}$$

The sign of $\partial_k \pi^*_{M\_d}$ is determined by $b_1(b_2 + d) - dn(b_1 + d)$ and $b_1(2b_2 + d - dn) + n(b_2^2 - d^2 - b_2\, d(n+1))$, and this is easily proved to be positive because $b_1 \geq dn$ and $b_2 \geq dn$. Therefore, $\pi^*_{M\_d}$ is an increasing function of k.

Next, we compare the expected profits between the non-participating and participating retailer. When $w < p_0$, $\pi^{N*}_{R\_d}(k) > \pi^{S*}_{R\_d}(k+1)$ (see Appendix 3); and $w = p_0$, due to mathematical complexity, we use numerical simulation to obtain similar results. The following Figure 2 shows the function of $\pi^{N*}_{R\_d}(k) - \pi^{S*}_{R\_d}(k+1)$ under different parameter combinations.

Based on the above results and given the set of retailers (K) participating in vertical information sharing, the equilibrium profit in stage one has the following properties:

(1) The manufacturer expected profit $\pi^*_{M\_d}$ is an increasing function of k. The more retailers share information, the better it is for the manufacturer. At the same time, with a decreasing function $\pi^*_{M\_d}$ (s_0) and more information sharing, the manufacturer will earn higher profit.

(2) The result $\pi^{N*}_{R\_d}(k) > \pi^{S*}_{R\_d}(k+1)$ indicates the non-participating retailer achieving more profit than retailers participating in private information sharing. As such, the retailer's best response is not to participate in vertical information sharing.

Using the above methods, we also study the situation of a single-channel structure and obtain similar equilibrium outcome.

In summary, vertical information sharing is always beneficial for the manufacturer but detrimental to the retailers in both single- and dual-channel structures, which indicates the retailers have no incentive to share information with the manufacturer. This implies the retailers are still unwilling to share information even if the direct channel is shut down. Thus far, we have shown that the change of channel structure does not affect the decision of the retailers to share information.

4.4 *Managerial insights for vertical information sharing*

From our analysis, we observe that non-participating retailers can infer demand information of the manufacturer from the wholesale price, and participating retailers do not obtain additional benefits from information sharing. This is the negative outcome of the 'leakage effect'. In a dual-channel structure, vertical information sharing can cause an increase in the direct price and wholesale price, which then reduces direct demand. The total effect for the retailers' profits is negative. Therefore, not disclosing information is the only Bayesian Nash equilibrium. By comparing the outcomes in different channel structures, we do not find any differences in outcome. This indicates the manufacturer's channel decision can be made without the consideration of information sharing.

In a dual-channel supply chain with uncertain demand, none of the retailers is willing to share its private information with the manufacturer; while the manufacturer always benefits from more vertical information sharing. Compared with the results in a dual-channel structure, closing the direct channel does not change the willingness of retailers to share information.

5. Horizontal information sharing

We have numerous combinations for horizontal information sharing between retailers. As such, we introduce the concept of partial sharing to simplify the analysis. We assume there are k ($k \geq 2$) retailers which form an information-sharing alliance and examine whether the participating retailers want to leave the system or the non-participating retailers would like to join.

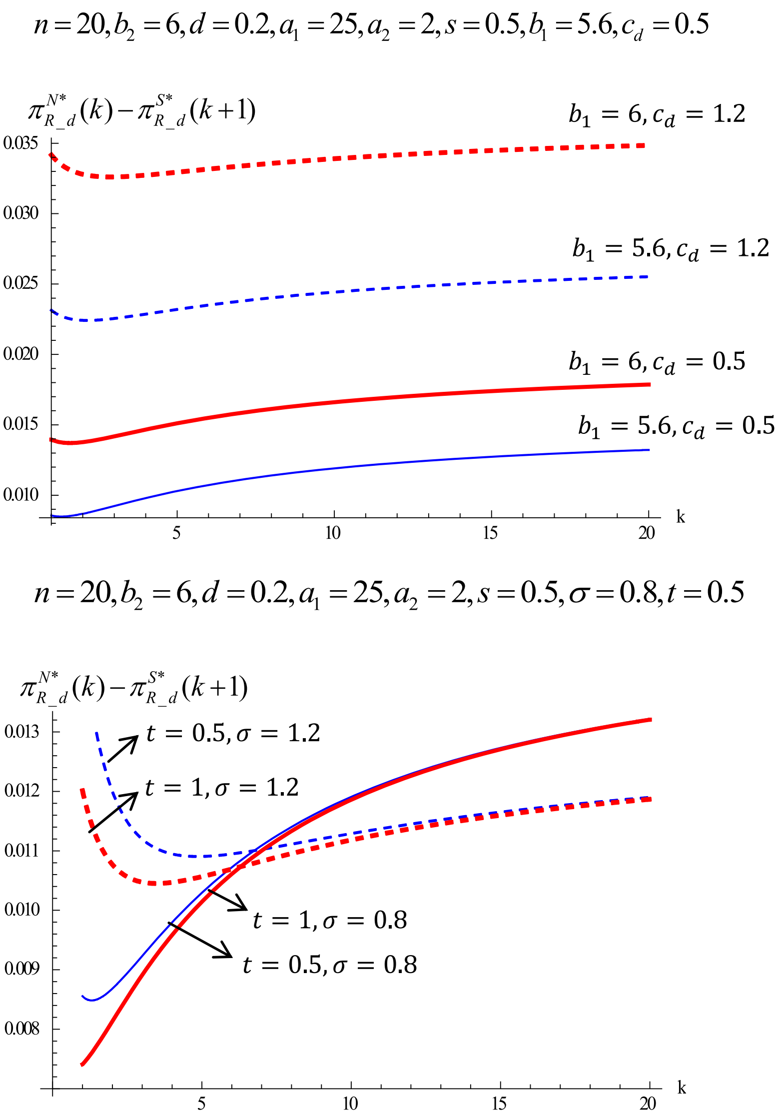


Figure 2. When $w = p_0$, the value of $\pi^{N*}_{R\_d}(k) - \pi^{S*}_{R\_d}(k+1)$ under different parameter combinations.

5.1 *Equilibrium outcome of horizontal sharing*

The manufacturer only has access to its private information set Y_0 in making decisions, and the wholesale price w is a function based on the set Y_0. Similar to the analysis in vertical sharing, we assume that w is a strictly increasing function of Y_0 and this is verified by the equilibrium outcome. Due to the information leakage effect, every retailer can infer the manufacturer's information Y_0.

Every retailer has its private demand forecast and chooses whether to join the information-sharing alliance. The information set is Y_k for a participating retailer and $\{Y_0, Y_i\}$for a non-participating retailer. Thus, we can solve the Bayesian Nash equilibrium (see the detailed proof in Appendix 4) and obtain the following outcomes:

Given the set of participating retailers (K) in a horizontal information sharing setting, the equilibrium profit in stage one has the following properties:

(1) The manufacturer expected profit $\pi^*_{M\_d}$ is uncorrelated with k, and remains the same as the non-sharing situation.

(2) These expected profits for participating and non-participating retailers, $\pi^S_{R\_d\_\mathrm{h}}(k) > \pi^N_{R\_d\_\mathrm{h}}(k-1)$, $\pi^N_{R\_d\_\mathrm{h}}(k) < \pi^S_{R\_d\_\mathrm{h}}(k+1)$, indicate that a non-participating retailer would like to join the information alliance in order to harvest additional profit, and no participating retailer would want to quit the alliance. As such the retailer's best response is to participate in horizontal information sharing.

Using the above approach, we also analyse the same issues in a single retail channel and obtain similar results: horizontal information sharing is always beneficial for the participating retailers and does not affect the manufacturer's expected profit.

In the Stackelberg (1934) model, the manufacturer makes the wholesale price decision without any information from the downstream retailers' information-sharing alliance. In addition, the information set which is used to decide the optimal wholesale price and direct price is the same as the non-sharing situation. As a result, the manufacturer's prices and the total expected profit are not affected by horizontal information sharing. By comparing the outcomes in dual-channel structures, we find the basic results are the same indicating the manufacturer's channel decision can be made without considering information sharing.

5.2 *Managerial insights for horizontal information sharing*

In a dual-channel supply chain with uncertain demand, all participating retailers are willing to share private information with each other; while the manufacturer does not always benefit from horizontal information sharing. Retailers participating in horizontal information sharing achieve higher profit margin when faced with severe price competition. When the manufacturer has less information, the participating retailer will achieve a higher profit margin. Compared with the results in a dual-channel structure, closing the direct channel does not change the equilibrium outcome of information sharing.

In a price-competitive environment, the retailers are more willing to share private information with others. The reason is that sharing information can increase the correlation relationship between the retailers to improve the expected profits. The results are consistent with prior research (Li 1985; Gal-Or 1986; Raith 1996) on horizontal information sharing in an oligopoly market.

6. Channel choice

According to the earlier information sharing results, retailers have no incentive to share information vertically in a free market. At the same, the manufacturer's expected profit is not affected by the horizontal information sharing, so the manufacturer can choose the channel structure as if there were no information sharing. Next, we examine the conditions under which the manufacturer should keep or close the direct channel in a dual-channel structure by comparing the expected profits in different channel structures.

To simplify the discussion, we assume the existing retail channel will not be shut down and there is no retailer deciding to drop out of the supply chain. The price sensitivity b_1 in the direct channel and the price sensitivity b_2 in the retail channel are not the same. However, for different channel structures, the price sensitivities of the retail channel, that is, b in the single channel and b_2 in the dual channel can be the same. Then we have $b_1 \geq dn$ and $b = b_2 > dn$. To explore the impact of uncertainty on the channel choice, we first discuss the situation with demand certainty and then extend the analysis to the demand uncertainty scenario. When solving the manufacturer's optimisation problem, there are two situations, $w < p_0$ and $w = p_0$. We focus on the former situation and examine the differences in the latter situation.

6.1 *Demand certainty situation*

Using the same method for the information sharing analysis under demand uncertainty, we examine the demand certainty situation.

(1) In a dual-channel structure with demand certainty, the equilibrium outcome is as follows:

The wholesale price is

$$w^* = \frac{a_2 b_1 + a_1 d}{2b_1(b_2 + d) - 2d(b_1 + d)n}.$$

The direct sale price is

$$p_0^* = \frac{(a_1 + b_1 c_d)(b_2 + d) - d(a_1 - a_2 + c_d(b_1 + d))n}{2b_1(b_2 + d) - 2d(b_1 + d)n}.$$

The demand in the direct channel is

$$q_0^* = \frac{(a_1 - b_1 c_d)(2b_2 + d) + d(-a_1 + a_2 + c_d(b_1 + d))n}{2(2b_2 + d - dn)}.$$

The manufacturer profit is

$$\pi^*_{m\_d} = \frac{1}{4}\left(\frac{(-a_1 + a_2 + c_d(b_1 + d))^2}{b_1 + d} - \frac{(2b_2 + d)(a_2 + c_d d)^2}{d(2b_2 + d - dn)} - \frac{(b_2 + d)(a_2 b_1 + a_1 d)^2}{d(b_1 + d)(-b_1(b_2 + d) + d(b_1 + d)n)}\right).$$

For retailer i, the retail price is

$$p^*_i = \frac{1}{2}\left(\frac{a_2 + c_d d}{2b_2 + d - dn} + \frac{a_2 b_1 + a_1 d}{b_1(b_2 + d) - d(b_1 + d)n}\right),$$

the total retail demand is

$$D^*_r = n(p_i - w) = \frac{(a_2 + c_d d)n}{2(2b_2 + d - dn)}.$$

the retailer i's profit is

$$\pi^*_i = b_2(pi - w)^2 = \frac{b_2(a_2 + c_d d)^2}{4(2b_2 + d - dn)^2}.$$

Next, we present two prerequisite conditions:

(i) The direct demand needs to be positive, which ensures a profitable direct channel. This is a prerequisite condition for the manufacturer to make the channel choice.

$$q^*_0 > 0 \Rightarrow c_d < \frac{a_2 dn + a_1(2b_2 + d - dn)}{2b_2 b_1 + d(b_1 - (b_1 + d)n)},$$

(ii) Here, we discuss the situation that the direct price is higher than the wholesale price.

$$p^*_0 > w^* \Rightarrow c_d > \frac{a_2 b_1 - a_1 b_2 + dn(a_1 - a_2)}{b_1(b_2 + d) - dn(b_1 + d)}$$

The retailer i's profit is

$$\pi^*_i = \frac{a^2 b}{4(2b - dn + d)^2}$$

and the manufacturer profit is

$$\pi^*_{m\_s} = \frac{a^2 bn}{4(b + d - dn)(2b + d - dn)}$$

(2) In a demand certainty environment, the difference between the manufacturer profit in single retail channel and that in a dual channel is:

$$\Delta\pi = \pi^*_{m\_d} - \pi^*_{m\_s} = \frac{V + A_1 c_d + A_2 c_d^2}{4((b + d - dn)(b_1(b + d) - d(b_1 + d)n))(2b + d - dn)}, \quad \text{here } b_2 = b:$$

To simplify our expression here, we define the following temporary parameters.

$$A_1 = -2(b + d - dn)(b_1(b + d) - d(b_1 + d)n)(a_2 dn + a_1(2b + d - dn));$$

$$A_2 = (b + d - dn)(b_2(b + d) - d(b_1 + d)n)(2bb_1 + d(b_1 - (b_1 + d)n))$$

$$\begin{aligned} V = {} & 2a_1 a_2 dn(b + d - dn)(2b + d - dn) + a_1^2(b + d - dn)^2(2b + d - dn) + na_2^2(b + d - dn)(bb_1 + d^2 n) \\ & + a^2 b(-b_1(b + d) + d(b_1 + d)n)) \end{aligned}$$

When $V < 0$, we have $\Delta\pi < 0$, the manufacturer is worse off if it has a dual-channel structure.

When $V > 0$, and marginal direct channel cost is lower than a threshold ($c_d < \bar{c}_d$), we have $\Delta\pi < 0$, the manufacturer is better off (see Appendix 5).

As shown in Figure 3, whether the direct channel should be kept open depends on the manufacturer's direct sales ability. To benefit from a dual-channel system, the conditions $c_d < \bar{c}_d$ vand $V > 0$ should be met. When $c_d < \bar{c}_d$, we have a low marginal cost of direct sale, which affects the manufacturer's ability to sell in the direct channel. Thus, $c_d < \bar{c}_d$ is defined as the internal condition. Correspondingly, the other condition which is required to be met, $V > 0$, contains the information of price sensitivities in different channels and degree of consumers' preference of the direct channel. This condition reflects the requirements of the external market environment. As such we define $V > 0$ as representing the external condition. Next, we will discuss the detailed implication of the external condition $V > 0$.

(3) We assume the total base demands are the same in a single-channel or dual-channel structure. The original base demands a_1, a_2 in a dual-channel structure and demand a in a single-channel structure satisfy the equation: $an = a_1 + a_2 n$.

Let $a_1 = \lambda\ an$ and $a_2 = (1\text{–}\lambda)a$. Here λ is the degree to which customers accept a direct channel as substitute for a retail channel. The greater the value of λ, the more market share the direct channel will have.

The degree of product substitutability is described by dn/b_1 and dn/b_2. A higher value would indicate stronger product substitutability.

We define the following variables: $e_1 = dn/b_1$, $e_2 = dn/b_2 = dn/b$, $0 < e_1, e_2 < 1$.

When these variables are substituted into the external condition, we get:

$$V = \frac{a^2 d^3 n^2}{e_1 e_2^3}(e_1 e_2^2(e_2 + 2n - e_2 n) + 2(1 + e_1(e_2 - 2))e_2(e_2(n-1) - n)n\lambda - (e_2(n-1) - n) \\ \times n(e_2 - e_1 e_2 + e_1(e_2 - 2)(e_2 - 1)n)\lambda^2)$$

From the two conditions $q_0 > 0$ and $p_0 > w$, we derive another constraint:

$$T = -(b_1(b+d) - d(b_1+d)n)(a_2 dn + a_1(2b + d - dn)) + (-a_1 b + a_2 b_1 + (a_1 - a_2)d \times n) \\ (2bb_1 + d(b_1 - (b_1 + d)n)) < 0$$

When we combine the two conditions stated above, the manufacturer has to meet the following external conditions to enable it to be better off with a dual-channel system (see Appendix 6):

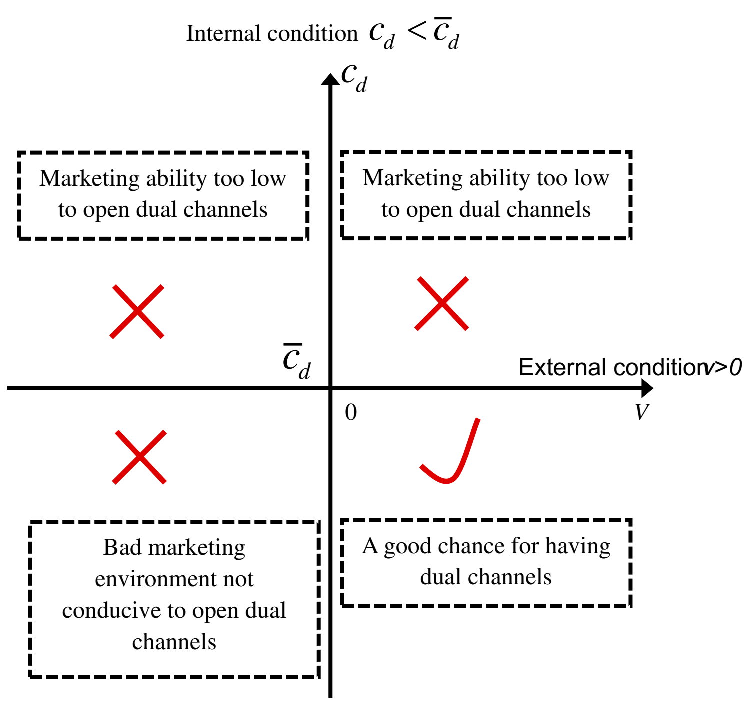


Figure 3. Two conditions for dual-channel structure.

When $e_1 \in \bar{\bar{e}}_1$, or $e_1 \in (\bar{e}_1, \bar{\bar{e}}_1)$ & $e_2 \in (0, \bar{e}_2)$, V is positive with $\lambda \in (\lambda^*, 1)$;
When $e_1 \in (0, \bar{e}_1)$, or $e_1 \in (\bar{e}, \bar{\bar{e}}_1)$ & $e_2 \in (\bar{e}_2, 1)$, V is positive with $\lambda \in (\max[\lambda^*, \lambda_2], 1)$
From a managerial perspective, the external condition always holds when:

(i) Price sensitivity in direct channel b_1 is small, or price sensitivity in retail channel b_2 is large, and degree of consumer preference for direct channel λ is not too small, or
(ii) Price sensitivity in direct channel b_1 is large, or price sensitivity in retail channel b_2 is small, and degree of consumer preference for direct channel λ is sufficiently large.

Figure 4 presents a simulation example to illustrate the range of parameters required to meet the external conditions and the constraint $T<0$. From the numerical simulation, we can see the above results hold. In order to guarantee $V>0$ and $T<0$ to hold simultaneously, the value of e_1 and λ should be sufficiently large and e_2 should be quite small.

6.2 *Demand uncertainty scenario*

Next we examine the channel choice decision under demand uncertainty. In the previous section, we have obtained the manufacturer expected profit in a dual-channel structure as follows:

$$\pi^*_{m\_d} = \frac{1}{4(2b_2+d-dn)(b_1(b_2+d)-d(b_1+d)n)}\Big((a_1-b_1c_d)^2(b_2+d)(2b_2+d)+(a_2^2b_1b_2+2a_2d(-b_1c_d(b_2+d)$$
$$+a_1(2b_2+d))+d(-a_1^2(3b_2+2d)-b_1c_d^2(b_1+d)(3b_2+2d)+2a_1c_d(3b_1b_2+2(b_1+b_2)d+d^2)))n$$
$$+d^2(-a_1+a_2+c_d(b_1+d))^2n^2+\frac{((b_2+d)(2b_2+d)+b_2(b_1+d)n)t\sigma^2}{s+t}\Big)$$

The manufacturer expected profit in a single-channel structure is:

$$\pi^*_{m\_s} = \frac{bn(a^2+\frac{t\sigma^2}{s+t})}{4(b+d-dn)(2b+d-dn)}$$

(2) In a demand uncertainty environment, the difference between the manufacturer expected profit in a single retail channel and a dual channel is:

$$\Delta\tilde{\pi} = \pi^*_{m\_d} - \pi^*_{m\_s} = \frac{\tilde{V}+A_1c_d+A_2c_d^2}{4((b+d-dn)(b_1(b+d)-d(b_1+d)n))(2b+d-dn)}, \quad \text{where } b_2=b$$

To simplify our expression here, we define the following temporary parameters.

$$A_2 = (b+d-dn)(b_2(d+d)) - d(b_1+d)n)(2bb_1+d(b_1(b_1+d)n))$$
$$A_1 = -2(b+d-dn)(b_1(b+d)) - d(b_1+d)n) + a_1(2b+d-dn)$$

$$\tilde{V} = 2a_1a_2dn(b+d-dn)(2b+d-dn)+a_1^2(b+d-dn)^2(2b+d-dn)$$
$$+n(a_2^2(b+d-dn)(bb_1+d^2n)+a^2b(-b_1(b+d)+d(b_1+d)n))+\frac{((b_2+d)(2b_2+d)+b_2(b_1+d)n)t\sigma^2}{s+t}$$

When $\tilde{V}<0$, we have $\Delta\tilde{\pi}<0$, the manufacturer is worse off in a dual-channel structure.

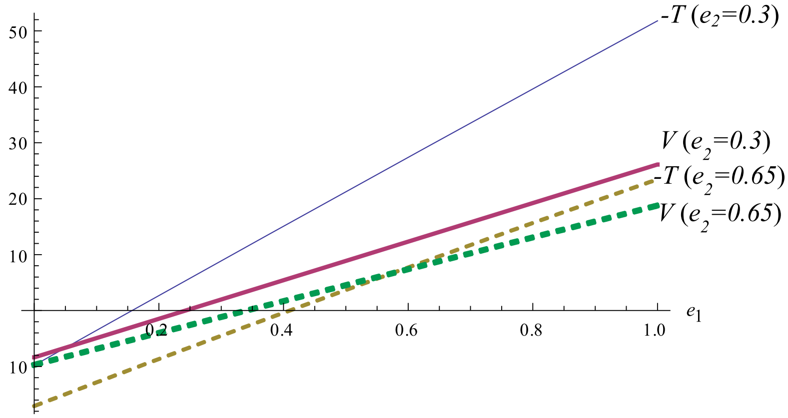
When $\tilde{V}>0$ and marginal direct channel cost is lower than a threshold ($c_d<\bar{c}_d^{\sigma}$), we have $\Delta\tilde{\pi}>0$, the manufacturer is better off (see Appendix 7).

As to the implication of the external condition of $\tilde{V}>0$, the conclusions are basically the same as the demand certainty situation.

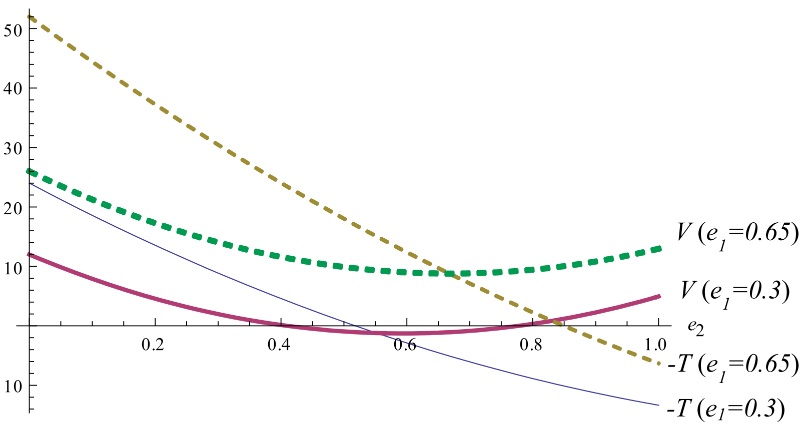
Next we discuss the impact of demand uncertainty.

$$\frac{((b_2+d)(2b_2+d)+b_2(b_1+d)n)t\sigma^2}{s+t} = 0$$

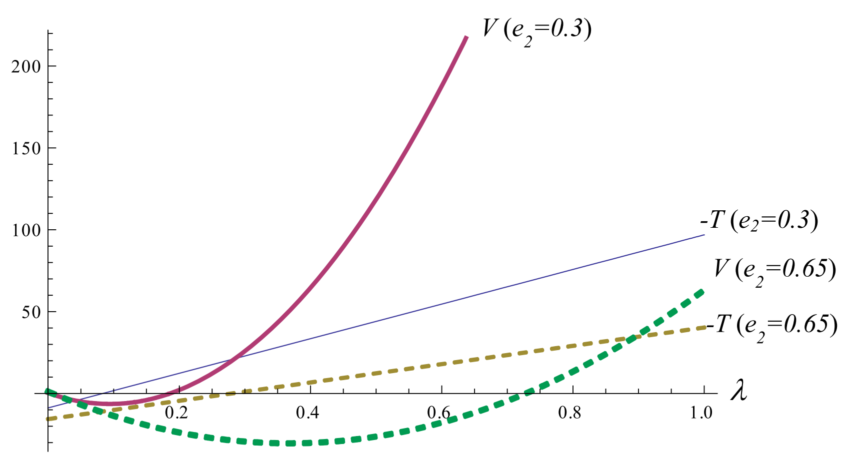
$$\Rightarrow \frac{((b_2+d)(2b_2+d)+b_2(b_1+d)n)t\sigma^2}{s+t} = \frac{((b_2+d)(2b_2+d)+b_2(b_1+d)n)\sigma^2}{s_0+1} = 0$$



(1) Given $n=20$ and $\lambda=0.05$, when $e_2=0.3$ or $e_2=0.65$, the change of $-T$ and V with respect to e_1



(2) Given $n=20$ and $\lambda=0.05$, when $e_1=0.3$ or $e_1=0.65$, the change of $-T$ and V with respect to e_2



(3) Given $n=20$ and $e_1=0.1$, the change of $-T$ and V with respect to λ

Figure 4. An illustrative simulation example for the effect of varying parameters.

When $\sigma=0$ or $s_0 \rightarrow \infty$, the uncertainty scenario degenerates into the certainty scenario. Another finding is that when the manufacturer has more information, the constraint for direct sale cost will be more relaxed.

6.3 *Scenario when the wholesale price is equal to direct channel price*

When the wholesale price is equal to the direct price, that is $w = p_0$, the condition $c_d < \frac{a_2 b_1 - a_1 b_2 + dn(a_1 - a_2)}{b_1(b_2+d) - dn(b_1+d)}$ should be met.

$\frac{a_2 b_1 - a_1 b_2 + dn(a_1 - a_2)}{b_1(b_2+d) - dn(b_1+d)} > 0 \Rightarrow \lambda < 1/(1 + n\frac{b_2 - dn}{b_1 - dn})$. This means that only when c_d and λ were sufficiently small, $w = p_0$ can be achieved. When the demand share and cost in direct channel are low, the manufacturer will reduce the direct price to the wholesale price.

The manufacturer expected profit in a dual-channel structure is as follows:

$$\begin{aligned}\pi^*_{m\_d} &= \mathrm{a}_2^2\mathrm{b}_2^2 n^2 + 2\mathrm{a}_2^2\mathrm{b}_2 dn^2 - 6\mathrm{a}_2\mathrm{b}_2^2\mathrm{c}_\mathrm{d} dn^2 + \mathrm{a}_2^2 d^2 n^2 + \mathrm{b}_2^2\mathrm{c}_\mathrm{d}^2 d^2 n^2 + 2\mathrm{a}_2\mathrm{c}_\mathrm{d} d^3 n^2 + 2\mathrm{b}_2\mathrm{c}_\mathrm{d}^2 d^3 n^2 + \mathrm{c}_\mathrm{d}^2 d^4 n^2 + 4\mathrm{a}_2\mathrm{b}_2\mathrm{c}_\mathrm{d} d^2 n^3 \\ &\quad + 2\mathrm{a}_2\mathrm{b}_1\mathrm{b}_2\mathrm{c}_\mathrm{d} n(2\mathrm{b}_2 + d - dn) - 4\mathrm{a}_1\mathrm{b}_2^2\mathrm{c}_\mathrm{d} n(2\mathrm{b}_2 + d - dn) - 2\mathrm{b}_1\mathrm{b}_2\mathrm{c}_\mathrm{d}^2 dn(2\mathrm{b}_2 + d - dn) \\ &\quad + 2\mathrm{a}_1 d(\mathrm{a}_2 + \mathrm{c}_\mathrm{d} d)n(2\mathrm{b}_2 + d - dn) - 2\mathrm{b}_1\mathrm{c}_\mathrm{d} d(\mathrm{a}_2 + \mathrm{c}_\mathrm{d} d)n(2\mathrm{b}_2 + d - dn) + \mathrm{a}_1^2(2\mathrm{b}_2 + d - dn)^2 \\ &\quad - 2\mathrm{a}_1\mathrm{b}_1\mathrm{c}_\mathrm{d}(2\mathrm{b}_2 + d - dn)^2 + \mathrm{b}_1^2\mathrm{c}_\mathrm{d}^2(2\mathrm{b}_2 + d - dn)^2 + \pi^*_{m\_d} \\ &= \frac{2\mathrm{a}_1\mathrm{b}_2 n(2\mathrm{b}_2 + d - dn)(\mathrm{a}_2 + \mathrm{c}_\mathrm{d} d(1+2n)) + \frac{(d + \mathrm{b}_2(2+n))^2(1+k+t)\delta^2}{1+k+s+t}}{4(2\mathrm{b}_2 + d - dn)(\mathrm{b}_1(2\mathrm{b}_2 + d - dn) - n(-\mathrm{b}_2^2 + d^2 + \mathrm{b}_2 d(1+n)))}\end{aligned}$$

In a demand uncertainty environment, the difference between the manufacturer expected profit in a single retail channel and a dual channel is:

$$\Delta\tilde{\pi} = \pi^*_{m\_d} - \pi^*_{m\_s} = \frac{M_1 c_d^2 + M_2 c_d + M_3}{4(2\mathrm{b}_2 + d - dn)}, \quad \text{where } M_1 = \frac{(d(\mathrm{b}_2 + d)n - \mathrm{b}_1(2\mathrm{b}_2 + d - dn))^2}{(\mathrm{b}_1(2\mathrm{b}_2 + d - dn) - n(-\mathrm{b}_2^2 + d^2 + \mathrm{b}_2 d(1+n)))}$$

$$M_2 = \frac{2\mathrm{a}_2 n(\mathrm{b}_1(\mathrm{b}_2 - d)(2\mathrm{b}_2 + d - dn) + dn(-3\mathrm{b}_2^2 + d^2 + 2\mathrm{b}_2 dn)) - 2\mathrm{a}_1(2\mathrm{b}_2 + d - dn)(\mathrm{b}_1(2\mathrm{b}_2 + d - dn) - n(-2\mathrm{b}_2^2 + d^2 + \mathrm{b}_2(d + 2dn)))}{\mathrm{b}_1(2\mathrm{b}_2 + d - dn) - n(-\mathrm{b}_2^2 + d^2 + \mathrm{b}_2 d(1+n))}$$

$$\begin{aligned}M_3 &= \frac{\mathrm{a}_2^2\mathrm{b}_2^2 n^2 + 2\mathrm{a}_2^2\mathrm{b}_2 dn^2 + \mathrm{a}_2^2 d^2 n^2 + 2\mathrm{a}_1\mathrm{a}_2\mathrm{b}_2 n(2\mathrm{b}_2 + d - dn) + 2\mathrm{a}_1\mathrm{a}_2 dn(2\mathrm{b}_2 + d - dn) + \mathrm{a}_1^2(2\mathrm{b}_2 + d - dn)^2}{\mathrm{b}_1(2\mathrm{b}_2 + d - dn) - n(-\mathrm{b}_2^2 + d^2 + \mathrm{b}_2 d(1+n))} \\ &\quad + \frac{(d + \mathrm{b}_2(2+n))^2(1+k+t)\delta^2}{(\mathrm{b}_1(2\mathrm{b}_2 + d - dn) - n(-\mathrm{b}_2^2 + d^2 + \mathrm{b}_2 d(1+n)))(1+k+s+t)} - \frac{\mathrm{b}_2 n((\mathrm{a}_1 + \mathrm{a}_2)^2 + t\delta^2/(s+t))}{\mathrm{b}_2 + d - dn}\end{aligned}$$

Due to calculation complexity, we use numerical simulation to analyse the condition for the manufacturer to choose a dual channel. Comparing with the results in the situation $w < p_0$, the only difference is that the price sensitivity in the retail channel should be low instead of high. Figure 5 shows the influence of price sensitivity in retail channel b_2 on the manufacturer's expected profit. When b_2 goes up, $\Delta\tilde{\pi} = \pi^*_{m\_d} - \pi^*_{m\_s}$ decreases and the consumer preference degree λ and cost c_d in direct channel should be low to meet the constraint of $p_0 = w$. Therefore, when the wholesale price is equal to the direct price, it is more tempting for a manufacturer to use a dual-channel structure if the price sensitivity in the retail channel is lower.

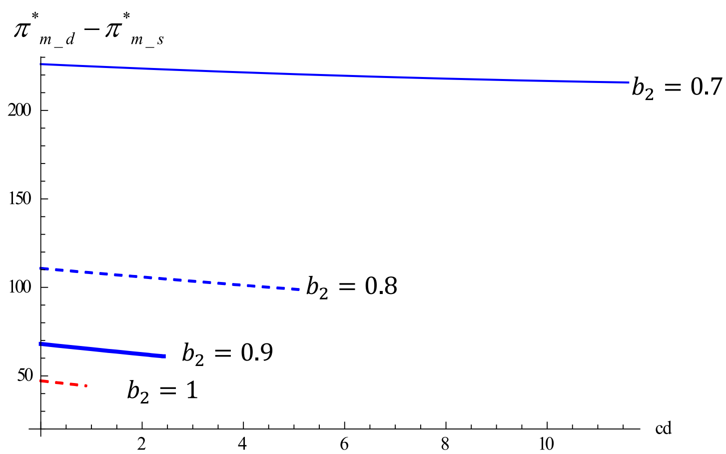
6.4 *Managerial insights for channel choice*

In a dual-channel supply chain with demand certainty, there are internal and external conditions that must be met if the manufacturer wants to be better off after opening the direct channel.

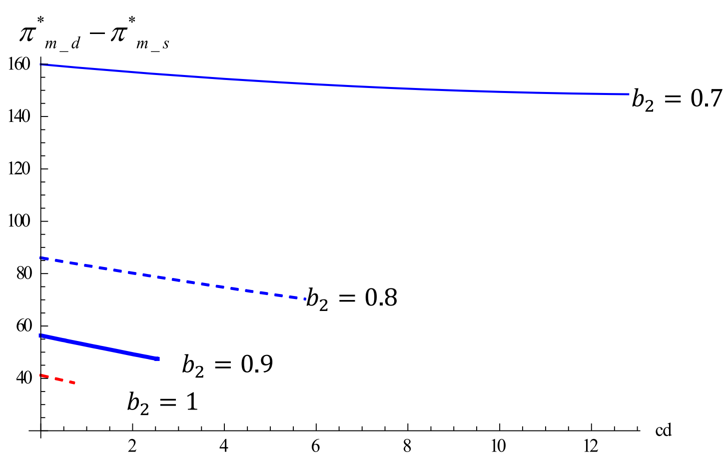
(1) The internal condition requires sufficiently high sales efficiency in the direct channel, which means the direct sale cost has to be relatively low.
(2) The external condition requires a favourable market environment: low price sensitivity in direct channel and sufficient direct demand market share.

After incorporating demand uncertainty to the model, our basic conclusions are similar. By comparison, we find that:

(1) Without demand uncertainty or any information sharing for the manufacturer, the uncertainty scenario degenerates into the certainty scenario.
(2) Demand uncertainty encourages the manufacturer to open a new direct channel. When the demand fluctuation increases, the internal conditions will be more relaxed, and it is more likely for the manufacturer to open the direct channel even if the cost is high.



(1) Given $n=3, d=0.2, a=10, s=0.5, \sigma=0.8, \lambda=0.12, b_1=0.8$



(2) Given $n=3, d=0.2, a=10, s=0.5, \sigma=0.8, \lambda=0.18, b_1=0.9$

Figure 5. Static analysis of price sensitivity b_2 under different parameter combinations.

The internal condition is for the manufacturer's direct sales channel to be profitable otherwise the direct channel will become a burden for the manufacturer. The external condition is related to the market environment. A favourable external condition requires a low price sensitivity of the direct channel and high price sensitivity of the retail channel. Specially, when the wholesale price equals the direct price, the price sensitivity in retail channel should also be low.

When the brand matures, consumers have developed product loyalty and the price sensitivity towards the direct channel should be relatively low. If the direct channel price goes up slightly, the direct channel demand would not likely decrease, which will help the direct channel profit.

Once the direct channel is set up, the retailers have another competitor, which compels them to reduce the retail price. This will result in an increase in demand due to the greater price sensitivity in the retail channel. As such, the manufacturer profit in the retail channel would not be affected too much. All of these provide a good market environment for the manufacturer to set up the direct channel. Specially, when the cost and degree of consumer preference in the direct channel are sufficiently low, the wholesale price can be adjusted to equal the sale price in the direct channel. Thus the retailer will consider increasing the retail price, but the demand in retail channel will not decrease too much because of low price sensitivity in the retail channel. Even if the consumer preference degree for retail channel is high, the manufacturer can still earn a good profit overall. Therefore, it is still favourable for the manufacturer to choose the dual-channel structure.

To study the effects of the degree of consumer preference for direct channel over the retail channel (λ) more clearly we examine the relationship between λ and the direct sale price p^*_0, wholesale price w^*:

$\partial w^*/\partial\lambda = a(-b_1 + dn)\lambda/(2b_1(b_2 + d) - 2d(b_1 + d)n) < 0$, w^* is negatively correlated with λ.
$\partial p_0^*/\partial\lambda = an(b_2 - dn)/(2b_1(b_2 + d) - 2d(b_1 + d)n) > 0$, p^*_0 is positively related with λ.

When the value of λ is increased, the wholesale price will decrease and the direct sale price will increase. These weaken the channel competition and balance the manufacturer profits between the two channels. When the degree of consumer preference for direct channel over the retail channel (λ) goes up, the manufacturer can increase its profit in the direct channel; although the original retail profit will decrease, the total profit in the dual channel can exceed the single retail channel.

Compared with the demand certainty scenario, in the uncertainty scenario the greater the demand fluctuation, the greater is the effect on the retail channel, which directly reduces the retailers' orders, and thus the manufacturer's profit in the retail channel will be reduced. The direct channel has some advantages in gaining consumers' surplus compared to the retail channel. As such, opening up a direct channel becomes a strategic choice to reduce risk loss due to market uncertainty. When the demand fluctuation increases, the external conditions will be more relaxed, and it favours the manufacturer to adopt a dual-channel structure.

7. Conclusion

In a dual-channel supply chain with price competition, we developed a game theoretic formulation. Vertical information sharing can allow the direct price to increase, which reduces direct demand, but also increases the wholesale price, with the result that retailers' profits are decreased. Therefore, not disclosing information for the retailers is the only Bayesian Nash equilibrium, while the manufacturer always benefits from vertical information sharing. The managerial insight gained would enable the manufacturer to consider paying retailers a fee to obtain the demand information. In addition, compared with the results in a dual-channel structure, closing the direct channel does not change the outcome of information sharing.

With respect to horizontal information sharing, the retailers are willing to participate in sharing information, with the reasoning that sharing information can improve the relationship between retailers and this kind of association can bring additional profit for participating retailers. The results are consistent with prior research (Li 1985; Gal-Or 1986; Raith 1996) about horizontal sharing in an oligopoly market. In a dual-channel environment, we find the basic results are similar. This indicates the manufacturer's channel decision can be made without changing the behaviour of information sharing.

Since vertically retailers have no incentive to share information and horizontally the manufacturer's expected profit is not affected, irrespective of whether the retailers share information among each other, then the manufacturer can choose the channel structure as if there were no information sharing. We showed that there are internal and external conditions that have to be met if the manufacturer wants to be better off from using the dual-channel structure. The internal condition requires high sales efficiency in the direct channel, which means the direct sales price has to be relatively low. The external condition involves a favourable market environment characterised by low price sensitivity in the direct channel, high price sensitivity in the retail channel and a sufficiently large proportion of demand from direct sales. Specifically, under the arbitrage situation, there should be low price sensitivity in the retail channel. Compared with the situation of certain demand, demand uncertainty encourages the manufacturer to open a new direct channel. When the demand fluctuation increases, the internal conditions will be more relaxed, and it is more probable for the manufacturer to open the direct channel even if the direct cost is high. As such, opening a direct channel is a strategic choice to reduce the risk due to market uncertainty. The conclusions obtained here can provide managerial guidance for the manufacturer to decide whether to open a new direct channel or close the existing direct channel in order to maximise the expected profit. We believe our research has contributed towards the understanding of information sharing and channel choice in a dual-channel supply chain with multiple retailers.

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Appendix 1: Lemma 1

First, for $i \in N\backslash K$: $E[Y_i|\{Y_j\}, j \in K_0] = E[E[Y_i|\theta, \{Y_j\}, j \in K_0]|\{Y_j\}, j \in K_0] = E[E[Y_i|\theta]|\{Y_j\}, j \in K_0] = E[\theta|\{Y_j\}, j \in K_0]$

We also know $E[\theta|\{Y_j\}, j \in K_0] = E[E[\theta|\{Y_m\}, m \in N_0]|\{Y_j\}, j \in K_0]$

According to the above results, we find that $E[\theta|\{Y_j\}, j \in K_0]$ is a linear combination of Y_j, $j \in K$.

Then we assume the linear combination has the following format:

$$E[\theta|\{Y_j\}, j \in K_0] = \alpha_0 Y_0 + \sum_{j \in K} \alpha_j Y_j$$

Thus, the problem is converted into finding the value of α_j to minimise $E[(\theta - \alpha_0 Y_0 - \sum_{j \in K} \alpha_j Y_j)^2]$.

$$E\left[(\theta - \alpha_0 Y_0 - \sum_{j \in K} \alpha_j Y_j)^2\right] = E\left[\theta^2 + \alpha_0^2 Y_0^2 + \left(\sum_{j \in K} \alpha_j Y_j\right)^2 - 2\alpha_0 \theta Y_0 - 2\theta \sum_{j \in K} \alpha_j Y_j + 2\alpha_0 Y_0 \sum_{j \in K} \alpha_j Y_j\right]$$

It is known that $E[\alpha_0^2 Y_0^2] = \alpha_0^2 \sigma^2 (1 + s_0)$, $E[2\alpha_0 \theta Y_0] = 2\alpha_0 \sigma^2$, $E[2\theta \sum_{j \in K} \alpha_j Y_j] = 2\sigma^2 \sum_{j \in K} \alpha_j$, $E[2\alpha_0 Y_0 \sum_{j \in K} \alpha_j Y_j] = 2\alpha_0 \sigma^2 \sum_{j \in K} \alpha_j$, $E[(\sum_{j \in K} \alpha_j Y_j)^2] = \sigma^2 (1 + s) \sum_{j \in K} \alpha_j^2 + \sigma^2 \sum_{i \neq j} \alpha_i \alpha_j$.

Hence,

$$E\left[(\theta - \alpha_0 Y_0 - \sum_{j \in K} \alpha_j Y_j)^2\right] = \sigma^2 + \alpha_0^2 \sigma^2 (1 + s_0) - 2\alpha_0 \sigma^2 - 2\sigma^2 \sum_{j \in K} \alpha_j + 2\alpha_0 \sigma^2 \sum_{j \in K} \alpha_j + \sigma^2 (1 + s) \sum_{j \in K} \alpha_j^2 + \sigma^2 \sum_{i \neq j} \alpha_i \alpha_j$$

$$\text{F. O. C: } s_0 \alpha_0 = s\alpha_i = 1 - \sum_{J \in K} \alpha_j - \alpha_0$$
$$\Rightarrow \alpha_0 = \left(\frac{s_0}{s} k + s_0 + 1\right)^{-1}, \quad \alpha_i = \left(k + \frac{s}{s_0} + s\right)^{-1}$$

Finally, we get $E[\theta|Y_j, j \in K_0] = (\frac{s_0}{s} k + s_0 + 1)^{-1} Y_0 + (k + \frac{s}{s_0} + s)^{-1} \sum_{j \in K} Y_j = \frac{t}{k+s+t} Y_0 + \frac{1}{k+s+t} \sum_{j \in K} Y_j$. Therefore $t \equiv s/s_0$.

Appendix 2: Stage three outcome in vertical sharing part

The first-order condition is: $2b_2 p_i^* = a_2 + E[\theta|\{Y_j\}, w] + dp_0 + d \sum_{j \neq i, j=1}^{n} E[p_j^*|\{Y_j\}, w] + b_2 w$.

Based on this, the retail price can be solved as follows.

Assume that $p_i^* = A^i + \sum_{j \in K_0} B_j^i Y_j + C^i Y_i$,

We find the solution has the following format:

When, $p_i^*(Y_j, j \in K_0) = \frac{1}{2b_2 - dn + d}(a_2 + b_2 \omega + dp_0 + \xi_1 \sum_{j \in K} Y_j + \xi_2 Y_0)$

When $i \in N\backslash K$, $p_i^*(Y_i, w) = \frac{1}{2b_2 - dn + d}(a_2 + b_2 \omega + dp_0 + \eta_1 (\sum_{j \in K} Y_j + tY_0) + \eta_2 Y_i)$

(1) When $i \in K$

$$\begin{aligned} 2b_2 p_i^* &= a_2 + b_2 \omega + dp_0 + E[\theta|Y_{K_0}, \omega] + d \sum_{l \neq i} E[p_l^*(Y_l, \omega)|Y_{K_0}, \omega] \\ &= a_2 + b_2 \omega + dp_0 + E[\theta|Y_{K_0}, \omega] + d \sum_{l \in K, l \neq i} E[p_l^*(Y_l, \omega)|Y_{K_0}, \omega] + d \sum_{l \notin K, l \neq i} E[p_l^*(Y_l, \omega)|Y_{K_0}, \omega] \end{aligned}$$

According to Lemma 1, we know that for a retailer who shares private information, $E[\theta|Y_{K_0}, \omega] = (\frac{t}{k+s+t} Y_0 + \frac{1}{k+s+t} \sum_{j \in K} Y_j)$, here $Y_{k_0} = \{Y_j\}_{j \in K_0}$ $\sum_{l \neq i, l \in K} E[p_l^*(Y_l, \omega)|Y_{K_0}.] = (k-1) p_i^*(Y_j, j \in K_0)$ $\sum_{l \neq i, l \notin K} E[p_l^*(Y_l, \omega)|Y_{K_0}] = \sum_{l \notin K} E[\frac{1}{2b_2 - dn + d}(a_2 + b_2 \omega + dp_0 + \eta_1 (\sum_{j \in K} Y_j + tY_0) + \eta_2 Y_l)|Y_{K_0}] = \frac{1}{2b_2 - dn + d} \sum_{l \notin K} [a_2 + b_2 \omega + dp_0 + \eta_1 (\sum_{j \in K} Y_j + tY_0) + \eta_2 (\frac{t}{k+s+t} Y_0 + \frac{1}{k+s+t} \sum_{j \in K} Y_j)] = \frac{n-k}{2b_2 - dn + d} (a_2 + b_2 \omega + dp_0 + \eta_1 (\sum_{j \in K} Y_j + tY_0) + \eta_2 (\frac{t}{k+s+t} Y_0 + \frac{1}{k+s+t} \sum_{j \in K} Y_j)) \rightarrow 2b_2 p_i^* = a_2 + b_2 \omega + dp_0 + (\frac{t}{k+s+t} Y_0 + \frac{1}{k+s+t} \sum_{j \in K} Y_j)$ $+ d(k-1) p_i^* + \frac{d(n-k)}{2b_2 - dn + d} (a_2 + b_2 \omega + dp_0 + \eta_1 (\sum_{j \in K} Y_j + tY_0) + \eta_2 (\frac{t}{k+s+t} Y_0 + \frac{1}{k+s+t} \sum_{j \in K} Y_j))$

Hence we get:

$$\frac{(2b_2-dk+d)}{2b_2-dn+d}\left(a_2+b_2\omega+dp_0+\xi_1\sum_{j\in K}Y_j+\xi_2Y_0\right)=a_2+b_2\omega+dp_0+\left(\frac{t}{\mathrm{k+s+t}}Y_0+\frac{1}{k+s+t}\sum_{j\in K}Y_j\right)$$
$$+\frac{d(n-k)}{2b_2-dn+d}\left(a_2+b_2\omega+dp_0+\eta_1\left(\sum_{j\in K}Y_j+tY_0\right)\right.$$
$$\left.+\eta_2\left(\frac{t}{\mathrm{k+s+t}}Y_0+\frac{1}{k+s+t}\sum_{j\in K}Y_j\right)\right)$$

Comparing the coefficients on the two sides of the above equation, we get:

$$(2b_2+d-dk)\xi_1==\frac{2b_2+d-dn}{k+s+t}+d(-k+n)\eta_1+\frac{d(-k+n)\eta_2}{k+s+t}$$

$$(2b_2+d-dk)\xi_2==\frac{(2b_2+d-dn)t}{k+s+t}+\frac{d(-k+n)t\eta_2}{k+s+t}+d(-k+n)t\eta_1$$

(2) When $i\in N\backslash K$

$$\begin{aligned}2b_2p_i^*&=a_2+b_2\omega+dp_0+E[\theta|\{Y_i,Y_{K_0}\},\omega]+d\sum_{l\neq i}E[p_l^*(Y_l,\omega)|\{Y_i,Y_{K_0}\},\omega]\\&=a_2+b_2\omega+dp_0+E[\theta|\{Y_i,Y_{K_0}\},\omega]+d\sum_{l\in K,l\neq i}E[p_l^*(Y_l,\omega)|\{Y_i,Y_{K_0}\},\omega]+d\sum_{l\notin K,l\neq i}E[p_l^*(Y_l,\omega)|\{Y_i,Y_{K_0}\},\omega]\end{aligned}$$

According to Lemma 1, we know that for a retailer i who does not share private information,

$$E[\theta|\{Y_i,Y_{K_0}\},\omega]=\frac{1}{k+s+t+1}tY_0+\frac{1}{k+s+t+1}\left(Y_i+\sum_{j\in K}Y_j\right)\sum_{l\in K}E[p_l^*(Y_l,\omega)|\{Y_i,Y_{K_0}\},w]=kp_{i_0}^*,\quad i_0\in K$$

$$\begin{aligned}&\sum_{l\notin K,l\neq i}E[p_l^*(Y_l,\omega)|\{Y_i,Y_{K_0}\},\omega]\\&=\sum_{l\notin K,l\neq i}E[\frac{1}{2b_2-dn+d}(a_2+b_2\omega+dp_0+\eta_1(\sum_{j\in K}Y_j+tY_0)+\eta_2Y_l)|\{Y_i,Y_{K_0}\},\omega]\\&=\frac{1}{2b_2-dn+d}\sum_{l\notin K,l\neq i}[a_2+b_2\omega+dp_0+\eta_1(\sum_{j\in K}Y_j+tY_0)+\eta_2(\frac{1}{k+s+t+1}(tY_0+\sum_{j\in K}Y_j)+\frac{1}{k+s+t+1}Y_i)]\\&=\frac{n-k-1}{2b_2-dn+d}(a_2+b_2\omega+dp_0+\eta_1(\sum_{j\in K}Y_j+tY_0)+\eta_2(\frac{1}{k+s+t+1}(tY_0+\sum_{j\in K}Y_j)+\frac{1}{k+s+t+1}Y_i))\end{aligned}$$

Hence,

$$\begin{aligned}&\frac{2b_2}{2b_2-dn+d}(a_2+b_2\omega+dp_0+\eta_1\left(\sum_{j\in K}Y_j+tY_0\right)+\eta_2Y_i)\\&=a_2+b_2\omega+dp_0+\frac{1}{k+s+t+1}\left(tY_0+\sum_{j\in K}Y_j\right)+\frac{1}{k+s+t+1}Y_i\\&+\frac{dk}{2b_2-dn+d}\left(a_2+b_2\omega+dp_0+\xi_1\sum_{j\in K}Y_j+\xi_2Y_0\right)+\frac{d(n-k-1)}{2b_2-dn+d}\left(a_2+b_2\omega+dp_0+\eta_1\left(\sum_{j\in K}Y_j+tY_0\right)\right.\\&\left.+\eta_2\left(\frac{1}{k+s+t+1}\left(tY_0+\sum_{j\in K}Y_j\right)+\frac{1}{k+s+t+1}Y_i\right)\right)\end{aligned}$$

Comparing the coefficients on the two sides of the above equation, we get:

$$2b_2\eta_1=\frac{2b_2+d-dn}{1+k+s+t}+dk\xi_1+d(-1-k+n)\eta_1+\frac{d(-1-k+n)\eta_2}{1+k+s+t}$$

$$2b_2\eta_2 = \frac{2b_2 + d - dn}{1+k+s+t} + \frac{d(-1-k+n)\eta_2}{1+k+s+t}$$

Solving the four simultaneous equations, we obtain the following:

$$\xi_1 = \frac{1}{k+s+t}, \quad \xi_2 = t\xi_1, \quad \eta_1 = (1-\eta_2)\frac{1}{k+s+t}, \quad \eta_2 = \frac{2b-dn+d}{2b(k+s+t+1)-d(n-k-1)}$$

Appendix 3: Stage one outcome in vertical sharing part

For retailer i, when $i \in K$

$$\pi^S_{R\_d}(k) = \frac{b_2}{4(2b_2 - dn + d)^2}\left(a_2^2 + 4a_2c_dd + c_d^2d^2 + \frac{k+t}{k+s+t}\sigma^2\right)$$

When $i \notin K$

$$\pi^N_{R\_d}(k) = \frac{b_2}{4(2b_2 - dn + d)^2}\left(a_2^2 + 4a_2c_dd + c_d^2d^2 + \frac{k+t}{k+s+t}\sigma^2 + 4\eta_2^2\sigma^2 s\frac{k+t+s+1}{k+s+t}\right)$$

We need to check the sign of $\pi^N_{R\_d}(k) - \pi^S_{R\_d}(k+1)$, which is equivalent to checking the following expression (Expre):

$$\begin{aligned}\text{Expre} &= \frac{k+t}{k+s+t} - \frac{1+k+t}{1+k+s+t} + \frac{4(-2\text{b}_2 - d + dn)^2 s(1+k+s+t)}{(k+s+t)(4\text{b}_2 + d + 2\text{b}_2(-1+k) + d(-1+k) - dn + 2\text{b}_2 s + 2\text{b}_2 t)^2} \\ &= \frac{s(6\text{b}_2(1+k+s+t) + d(k(3-2n) - 3n - 2n(s+t) + 2(1+s+t)))(2\text{b}_2(1+k+s+t) + d(k-n-2kn-2n(s+t)+2(1+s+t)))}{(k+s+t)(1+k+s+t)(d(k-n) + 2\text{b}_2(1+k+s+t))^2}\end{aligned}$$

Substitute $b_2 \geq dn$ into the above condition and we obtain the following:

$$\text{Expre} \geq \frac{s(d(3n + k(3+4n) + 4n(s+t) + 2(1+s+t)))(d(k+n+2(1+s+t)))}{(k+s+t)(1+k+s+t)(d(k-n) + 2\text{b}_2(1+k+s+t))^2} > 0$$

Hence, we find that $\pi^N_{R\_d}(k) - \pi^S_{R\_d}(k+1) > 0$.

Appendix 4: Horizontal sharing part

Horizontal Information Sharing

Stage three: retailers' price decision

In the last stage, given the horizontal information sharing alliance ($K \subseteq N$, $k = 2, \ldots n$), the wholesale price and direct sale price, the retailers need to set retail price to maximise the conditional expected profit:

$$E[\pi_i|\{Y_j\}, w] = \left\{a_2 + E[\theta|Y_i, w] - b_2p_i + dp_0 + d\sum_{j\neq i, j=1}^{n} E[p_j|Y_i, w]\right\}(p_i - w)$$

The first-order condition (FOC) is:

$$i \in K \quad a_2 + b_2w + E[\theta|Y_{K_0}] + d\sum_{j\neq i, j\in K} E[p_j|Y_{K_0}] + dp_0 + d\sum_{j\notin K} E[p_j|Y_{K_0}] = 2b_2p_i^*$$

$$i \notin K \quad a_2 + b_2w + E[\theta|Y_0, Y_i] + d\sum_{j\in K} E[p_j|Y_0, Y_i] + dp_0 + d\sum_{j\neq i, j\notin K} E[p_j|Y_0, Y_i] = 2b_2p_i^*$$

It is a static game with incomplete information in this stage. The equilibrium retail price every retailer sets is a monotone function of the information it owns, and satisfies the first-order condition.

Conclusion: Given the horizontal information sharing alliance ($K \subseteq N$, $k = 2, \ldots n$), the wholesale price w and the expectation that w is a strictly increasing function of Y_0, the sub-game in *Stage Three* has only the following Bayesian Nash equilibrium:

$$i \in K p_i^*(Y_{K_0}) = \frac{1}{2b_2 - dn + d}(a_2 + b_2 w + dp_0 + \xi_1 \sum_{j \in K} Y_j + \xi_2 Y_0)$$
$$i \notin K p_i^*(Y_{K_0}) = \frac{1}{2b_2 - dn + d}(a_2 + b_2 w + dp_0 + \eta_1 Y_i + \eta_2 Y_0)$$

$$x_2 = \frac{(t(-d^2(-1+k)(1+k-n) + 4b_2^2(1+s+t) - 2b_2 d(-2+n-s-t+k(k-n+s+t))))}{(4b_2^2(1+s+t)(k+s+t) - 2b_2 d(k+s+t)(-2+n-s-t+k(s+t)) + d^2(-k^2(s+t) - (-1+n)(s+t) + k(1+n(-1+s+t))))}$$

$$x_1 = \frac{((2b_2 + d - dn)(d + 2b_2(1+s+t)))}{(4b_2^2(1+s+t)(k+s+t) - 2b_2 d(k+s+t)(-2+n-s-t+k(s+t)) + d^2(-k^2(s+t) - (-1+n)(s+t) + k(1+n(-1+s+t))))}$$

$$\eta_2 = \frac{((2b_2 + d - dk)t(dk + 2b_2(k+s+t)))}{(4b_2^2(1+s+t)(k+s+t) - 2b_2 d(k+s+t)(-2+n-s-t+k(s+t)) + d^2(-k^2(s+t) - (-1+n)(s+t) + k(1+n(-1+s+t))))}$$

$$\eta_1 = \frac{((2b_2 + d - dn)(d - d(-1+k)(-1+s+t) + 2b_2(k+s+t)))}{(4b_2^2(1+s+t)(k+s+t) - 2b_2 d(k+s+t)(-2+n-s-t+k(s+t)) + d^2(-k^2(s+t) - (-1+n)(s+t) + k(1+n(-1+s+t))))}$$

Stage two: manufacturer's price decision

Given the retailers' equilibrium prices, the manufacturer sets the wholesale price and the direct sale price to maximise the following conditional expected profit:

$$\begin{aligned}
E[p_M|Y_{K_0}] &= E\left[w \sum_{i \in N} q_i^* + (p_0 - C_d)\left(a_1 + \theta - b_1 p_0 + d \sum_{i \in N} p_i^*\right)|Y_{K_0}\right] \\
&= b_2 w\left((-k+n)\left(-w + \frac{a_2 + dp_0 + b_2 w + \frac{(1+t)y_0\eta_1}{1+s+t} + y_0\eta_2}{2b_2 + d - dn}\right) + k(-w + \frac{a_2 + dp_0 + b_2 w + \frac{k(1+t)y_0 x_1}{1+s+t} + y_0 x_2}{2b_2 + d - dn})\right) \\
&+ (-c_d + p_0)\left(a_1 - b_1 p_0 + \frac{(1+t)y_0}{1+s+t}\right. \\
&\left.+ d\left(\frac{(-k+n)(a_2 + dp_0 + b_2 w + \frac{(1+t)y_0 h_1}{1+s+t} + y_0 h_2)}{2b_2 + d - dn} + \frac{k(a_2 + dp_0 + b_2 w + \frac{k(1+t)y_0 x_1}{1+s+t} + y_0 x_2)}{2b_2 + d - dn}\right)\right)
\end{aligned}$$

Conclusion: Given any information arrangement ($K \subseteq N$, $k = 2, \ldots, n$), the manufacturer's equilibrium prices are:

$$p_0 = \frac{((a_1 + b_1 c_d)(b_2 + d) - d(a_1 - a_2 + c_d(b_1 + d))n)(s+t) + (b_2 + d)ty_0}{2(b_1(b_2 + d) - d(b_1 + d)n)(s+t)}$$

$$w = \frac{(a_2 b_1 + a_1 d)(s+t) + (b_1 + d)ty_0}{2(b_1(b_2 + d) - d(b_1 + d)n)(s+t)}$$

Obviously, w is the monotone increasing function of Y_0; thus, we verify the expectation that the wholesale price is monotone related to the manufacturer's information.

Stage one: decision on horizontal information sharing

For all the retailers, the function of expected profit can be expressed as:

$$E[E[\pi_i|\{Y_j\}, w]] = b_2 \cdot E[(p_i - w)^2]$$

We define the expected profits of participating and non-participating retailers, respectively, as: $\pi^S_{R\_d\_\mathrm{h}}(k)$ and $\pi^N_{R\_d\_\mathrm{h}}(k)$

The change of expected profit after sharing information is:

$$\Delta\pi_{R\_\mathrm{h}}(k) = \pi^S_{R\_d\_\mathrm{h}}(k+1) - \pi^N_{R\_d\_\mathrm{h}}(k)$$

According to the numerical simulation results, $\Delta\pi_{R\_h}(k) > 0$ and is positively related with d/b_2ands_0. The ratio d/b_2 refers to product substitutability and competition in retail channel. When d/b_2 increases in value, horizontal sharing becomes more necessary and the profit margin increases with higher information sharing. When s_0 increases, which means the manufacturer has less accurate information, the participating retailer will increase its profit margin.

For the manufacturer, we define its expected profit as $\pi_{M\backslash d}(k)$:

$$E[p_{M\_d}] = \frac{\begin{array}{c}((a_1 - b_1c_d)^2(b_2+d)(2b_2+d) + (a_2^2b_1b_2 + 2a_2d(-b_1c_d(b_2+d) + a_1(2b_2+d))+ \\ d(-a_1^2(3b_2+2d) - b_1c_d^2(b_1+d)(3b_2+2d) + 2a_1c_d(3b_1b_2 + 2(b_1+b_2)d + d^2)))n \\ +d^2(-a_1+a_2+c_d(b_1+d))^2n^2)\end{array}}{((4(2b_2+d-dn)(b_1(b_2+d) - d(b_1+d)n)))}$$
$$+ \frac{(((a_1 - b_1c_d)(b_2+d)(2b_2+d) + (a_2(b_1b_2 + d(2b_2+d)) + d(-a_1(b_2+d) + c_d(b_1+d)(2b_2+d)))n)ty_0)}{(2(2b_2+d-dn)(b_1(b_2+d) - d(b_1+d)n)(s+t))}$$
$$+ \frac{(2b_2^2 + 3b_2d + d^2 + b_2(b_1+d)n)t^2y_0^2}{4(2b_2+d-dn)(b1(b_2+d) - d(b_1+d)n)(s+t)^2}$$

The expected profit is:

$$\begin{aligned} p_{M\_d}(k) &= E[E[p_{M\_d}|Y_{K_0}]] \\ &= \frac{((a_1 - b_1c_d)^2(b_2+d)(2b_2+d) + (a_2^2b_1b_2 + 2a_2d(-b_1c_d(b_2+d)}{4(2b_2+d-dn)(b1(b_2+d) - d(b_1+d)n)} + a_1(2b_2+d)) + d(-a_1^2(3b_2+2d) \\ &\quad - b_1c_d^2(b_1+d)(3b_2+2d) + 2a_1c_d(3b_1b_2 + 2(b_1+b_2)d + d^2)))n + d^2(-a_1+a_2+c_d(b_1+d))^2n^2 \\ &\quad + \frac{((b_2+d)(2b_2+d) + b_2(b_1+d)n)ts^2}{s+t}) \end{aligned}$$

The profit difference between participating and non-participating retailers is:

$$\Delta\pi_{R\_h}(k) = \pi^S_{R\_d\_h}(k+1) - \pi^N_{R\_d\_h}(k)$$

Specifically, when $w = p_0$, we can obtain the optimal prices:

$$p_0^* = w^* = \frac{\begin{array}{r} a_2b_2n + a_2dn - b_2c_ddn - c_dd^2n + a_2b_2ns + a_2dns - b_2c_ddns - c_dd^2ns + a_2b_2nt \\ + a_2dnt - b_2c_ddnt - c_dd^2nt + a_1(2b_2+d-dn)(1+s+t) + b_1c_d(2b_2+d-dn)(1+s+t) \\ + y_0(2b_2 + d - dn + 2b_2t + dt - dnt - b_2k\eta_1 - dk\eta_1 + b_2n\eta_1 + dn\eta_1 - b_2kt\eta_1 \\ -dkt\eta_1 + b_2nt\eta_1 + dnt\eta_1 - b_2k\eta_2 - dk\eta_2 + b_2n\eta_2 + dn\eta_2 - b_2ks\eta_2 - dks\eta_2 \\ +b_2ns\eta_2 + dns\eta_2 - b_2kt\eta_2 - dkt\eta_2 + b_2nt\eta_2 + dnt\eta_2 + b_2k^2\xi_1 + dk^2\xi_1 \\ +b_2k^2t\xi_1 + dk^2t\xi_1 + b_2k\xi_2 + dk\xi_2 + b_2ks\xi_2 + dks\xi_2 + b_2kt\xi_2 + dkt\xi_2) \end{array}}{2(b_1(2b_2+d-dn) - n(-b_2^2 + d^2 + b_2d(1+n)))(1+s+t)}$$

By numerical simulation, we find the above results remain the same.

Appendix 5: Profit comparison in demand certainty situation

$$\Delta\pi = \pi_{m-d} - \pi_{m-s} = \frac{V + A_1c_d + A_22c_d^2}{4((b+d-dn)(b_1(b+d) - d(b_1+d)n))(2b+d-dn)}, \quad \text{here } b_2 = b,$$

$$A_1 = -2(b+d-dn)(\mathrm{b}_1(b+d) - d(\mathrm{b}_1+d)n)(\mathrm{a}_2dn + \mathrm{a}1(2b+d-dn));$$

$$A_2 = (b+d-dn)(\mathrm{b}_1(b+d) - d(\mathrm{b}_1+d)n)(2b\mathrm{b}_1 + d(\mathrm{b}_1 - (\mathrm{b}_1+d)n));$$

$$\begin{aligned} V &= 2a_1a_2dn(b+d-dn)(2b+d-dn) + a_1^2(b+d-dn)^2(2b+d-dn) + n(a_2^2(b+d-dn)(bb_1+d^2n) \\ &\quad + a^2b(-b_1(b+d) + d(b_1+d)n)) \end{aligned}$$

It is easy to prove that: $A_2 > 0$, $A_1 < 0$, $\frac{A_1}{-2A_2} = \frac{a_2dn + a_1(2b+d-dn)}{2bb_1 + d(b_1 - (b_1+d)n)}$

$$\Delta\pi > 0 \Leftrightarrow \quad V + A_1c_d + A_2c_d^2 > 0$$

According the previous condition, we have

$$q_0 > 0 \Rightarrow c_d < \frac{a_2dn + a_1(2b + d - dn)}{2bb_1 + d(b_1 - (b_1 + d)n)}$$

$A_1c_d + A_2c_d^2 < 0$ and it is a decreasing function of c_d

To ensure the inequality $V + A_1c_d + A_2c_d^2 > 0$ has a solution, $V > 0$ must hold.

Define $V^* = \dfrac{(b + d - dn)(b_1(b + d) - d(b_1 + d)n)(a_2dn + a_1(2b + d - dn))^2}{2bb_1 + d(b_1 - (b_1 + d)n)}$.

When $V^* = V$, $V + A_1c_d + A_2c_d^2 = 0$ we have two similar roots.

Define c_d^1 as the smaller one in the real roots of $V + A_1c_d + A_2c_d^2 = 0$.

According to the property of quadratic function, we derive the following:

When $0 < V < V^*$, if $c_d < c_d^1$, then $\Delta\pi > 0$; if $c_d^1 < c_d < \dfrac{a_2dn + a_1(2b + d - dn)}{2bb_1 + d(b_1 - (b_1 + d)n)}$, then $\Delta\pi < 0$.

When $V > V^*$, $c_d < \dfrac{a_2dn + a_1(2b + d - dn)}{2bb_1 + d(b_1 - (b_1 + d)n)}$, $\Delta\pi > 0$

Here we introduce $\overline{c_d}$.

When $0 < V < V^*$, $\overline{c_d} = c_d^1$.

When $V > V^*$, $\overline{c_d} = \dfrac{a_2dn + a_1(2b + d - dn)}{2bb_1 + d(b_1 - (b_1 + d)n)}$

In summary, when $V > 0$ and $c_d < \overline{c_d}$, the manufacturer obtains more profit by opening the direct channel.

Appendix 6: External conditions in demand certainty situation

$$V = \frac{a^2d^3n^2(e_1e_2^2(e_2 + 2n - e_2n) + 2(1 + e_1(-2 + e_2))e_2(e_2(-1 + n) - n)n\lambda - (e_2(-1 + n) - n)n(e_2 - e_1e_2 + e_1(-2 + e_2)(-1 + e_2)n)\lambda^2)}{e_1e_2^3}$$

(1) Let $V' = (e_1e_2^2(e_2 + 2n - e_2n) + 2(1 + e_1(-2 + e_2))e_2(e_2(-1 + n) - n)n\lambda - (e_2(-1 + n) - n)n(e_2 - e_1e_2 + e_1(-2 + e_2)(-1 + e_2)n)\lambda^2) = B_0 + B_1 * \lambda + B_2 * \lambda^2$

$$B_2 = -(e_2(-1 + n) - n)n(e_2 - e_1e_2 + e_1(-2 + e_2)(-1 + e_2)n) > 0$$

$$B_1 = 2(1 + e_1(-2 + e_2))e_2(e_2(-1 + n) - n)n$$

$$B_0 = e_1e_2^2(e_2 + 2n - e_2n) > 0$$

It's easy to prove that $B_1/(-2B_2) < 1$.

The discriminant $\Delta = -4e_2^2n(-e_1e_2 + n + 2e_1(-2 + e_2)n)(e_2 + n - e_2n)(-n + e_2(-1 + e_1 + n))$

For $e_2 \in (0, 1)$, one possible root of$\Delta = 0$ is $\overline{e_2} = \dfrac{-n + 4e_1n}{e_1(-1 + 2n)}$.

Define $\bar{e}_1 = \frac{1}{4}, \bar{\bar{e}}_1 = \dfrac{n}{1 + 2n}$

(i) When $e_1 \in (\overline{e_1}, \bar{\bar{e}}_1)$, the above root exists.

And if $e_2 \in (0, \overline{e_2})$, $\Delta < 0$; if, $e_2 \in (\bar{e}_2, 1)\Delta > 0$.

(ii) When $e_1 \in (0, \overline{e_1})$, $\Delta > 0$; $.e_1 \in (\bar{\bar{e}}_1, 1).$, $\Delta < 0$. The above root does not exist.

When $e_1 \in (\bar{\bar{e}}_1, 1)$ or $e_1 \in (\overline{e_1}, \bar{\bar{e}}_1)$ and $e_2 \in (0, \overline{e_2})$, we have $\Delta < 0$, V is positive for $\lambda \in (0, 1)$.

When $e_1 \in (0, \overline{e_1})$ or $e_1 \in (\overline{e_1}, \bar{\bar{e}}_1)$ and $e_2 \in (\overline{e_2}, 1)$, we have $\Delta > 0$, $V = 0$ has two roots for $\lambda \in (0, 1)$, denoted as λ_1, λ_2

$$\lambda_1 = \frac{\begin{array}{c}(-2(1 + e_1(-2 + e_2))e_2(e_2(-1 + n) - n)n \\ -\sqrt{(4(1 + e_1(-2 + e_2))^2e_2^2(e_2(-1 + n) - n)^2n^2} \\ -4e_1e_2^2n(-e_2(-1 + n) + n)(e_2 - e_1e_2 + e_1(-2 + e_2)(-1 + e_2)n)(e_2 + 2n - e_2n)))\end{array}}{(2n(-e_2(-1 + n) + n)(e_2 - e_1e_2 + e_1(-2 + e_2)(-1 + e_2)n))}$$

$$\lambda_2 = \frac{\begin{array}{c}(-2(1+e_1(-2+e_2))e_2(e_2(-1+n)-n)n \\ +\sqrt{(4(1+e_1(-2+e_2))^2e_2^2(e_2(-1+n)-n)^2n^2} \\ -4e_1e_2^2n(-e_2(-1+n)+n)(e_2-e_1e_2+e_1(-2+e_2)(-1+e_2)n)(e_2+2n-e_2n)))\end{array}}{(2n(-e_2(-1+n)+n)(e_2-e_1e_2+e_1(-2+e_2)(-1+e_2)n))}$$

where $V>0$, $\lambda \in (0,\lambda_1)$ or $(\lambda_2, 1)$

(2) Next we examine another condition: $p_0 > w \Rightarrow c_d > \frac{a_2b_1 - a_1b_2 + dn(a_1 - a_2)}{b_1(b_2+d) - dn(b_1+d)} = \frac{a_2b_1 - a_1b + dn(a_1-a_2)}{b_1(b+d) - dn(b_1+d)}$

$$q_0 > 0 \Rightarrow c_d < \frac{a_2dn + a_1(2b+d-dn)}{2bb_1 + d(b_1-(b_1+d)n)} - (b_1(b+d) - d(b_1+d)n)(a_2dn + a_1(2b+d-dn))$$
$$+ (-a_1b + a_2b_1 + (a_1-a_2)dn)(2bb_1 + d(b_1-(b_1+d)n)) < 0$$

Let $b_1 = \frac{dn}{e_1}, b = \frac{dn}{e_2}, a_1 = \lambda na, a_2 = (1-\lambda)a$

$$\frac{ad^3n^2(-4e_1n^2\lambda + e_2n(2-3e_1+(-2+e_1(-1+3e_1+6n))\lambda) - e_2^2(-1+e_1+n)(1-\lambda+e_1(-2+\lambda+2n\lambda)))}{e_1^2e_2^2} < 0,$$

which is equivalent to

$$T = 2e_2n - 3e_1e_2n - e_2^2(-1+e_1+n) + 2e_1e_2^2(-1+e_1+n) + (-4e_1n^2 + e_2^2(-1+e_1+n) - e_1e_2^2(-1+e_1+n)$$
$$- 2e_1e_2^2n(-1+e_1+n) + e_2n(-2+e_1(-1+3e_1+6n)))\lambda < 0$$

We find that

$$(-4e_1n^2 + e_2^2(-1+e_1+n) - e_1e_2^2(-1+e_1+n) - 2e_1e_2^2n(-1+e_1+n) + e_2n(-2+e_1(-1+3e_1+6n))) < 0$$

and T is a decreasing function of λ.

Denote the root of $T=0$ as $\lambda^* = \frac{-e_2^2 + 3e_1e_2^2 - 2e_1^2e_2^2 - 2e_2n + 3e_1e_2n + e_2^2n - 2e_1e_2^2n}{\begin{array}{r}-e_2^2 + 2e_1e_2^2 - e_1^2e_2^2 - 2e_2n - e_1e_2n + 3e_1^2e_2n + e_2^2n + e_1e_2^2n \\ -2e_1^2e_2^2n - 4e_1n^2 + 6e_1e_2n^2 - 2e_1e_2^2n^2\end{array}}$

$$T < 0 \Leftrightarrow \lambda^* < \lambda < 1$$

Because $\frac{\partial \lambda^*}{\partial e_1} < 0$, λ^*decreases with an increase ine_1, T<0 is much easier to satisfy.

There exists e_1^* to enable $T = 0 \Leftrightarrow e_1^* < e_1 < 1$

$$e_1^* = \frac{1}{2(2e_2^2 - e_2^2\lambda + 3e_2n\lambda - 2e_2^2n\lambda)}(3e_2^2 + 3e_2n - 2e_2^2n - 2e_2^2\lambda + e_2n\lambda - e_2^2n\lambda + 4n^2\lambda - 6e_2n^2\lambda + 2e_2^2n^2\lambda - \sqrt{(-4(2e_2^2}$$
$$- e_2^2\lambda + 3e_2n\lambda - 2e_2^2n\lambda)(e_2^2 + 2e_2n - e_2^2n - e_2^2\lambda - 2e_2n\lambda + e_2^2n\lambda) + (-3e_2^2 - 3e_2n + 2e_2^2n + 2e_2^2\lambda - e_2n\lambda + e_2^2n\lambda$$
$$- 4n^2\lambda + 6e_2n^2\lambda - 2e_2^2n^2\lambda)^2))$$

Rewriting the condition T with respect toe_2, we have the following:

$$T = -4e_1n^2\lambda + e_2n(2-3e_1+(-2+e_1(-1+3e_1+6n))\lambda) + e_2^2(1-e_1-n)(1-\lambda+e_1(-2+\lambda+2n\lambda))$$

Define the root of $T=0$ as e_2^*

$$\frac{\partial T}{\partial e_2} = n(2-3e_1+(-2+e_1(-1+3e_1+6n))\lambda) + 2e_2(1-e_1-n)(1-\lambda+e_1(-2+\lambda+2n\lambda)) > 0$$

$$e_2^* = \frac{\begin{array}{c}-n(2-3e_1+(-2+e_1(-1+3e_1+6n))\lambda)+ \\ \sqrt{n^2(2-3e_1+(-2+e_1(-1+3e_1+6n))\lambda)^2 + 16e_1(1-e_1-n)n^2\lambda(1-\lambda+e_1(-2+\lambda+2n\lambda))}\end{array}}{2(1-e_1-n)(1-\lambda+e_1(-2+\lambda+2n\lambda))}$$

Therefore, $T<0 \Leftrightarrow 0 < e_2 < e_2^*$

(3) Finally, we combine the two conditions and obtain the following:

Substitute $\lambda = \lambda_1$ into the condition T, $T > 0$, which means $\lambda_1 < \lambda^*$ and we can abandon one possible result $\lambda \in (0,\lambda_1)$.

Substitute $\overline{e_2} = \frac{-n+4e_1n}{e_1(-1+2n)}$into the condition T and let $e_1 \in (\overline{e_1}, \overline{\overline{e_1}})$, we find T<0, that is, $\overline{e_2} < e_2^*$.

When or $e_1 \in (\overline{e_1}, \overline{\overline{e_1}})$ and $e_2 \in (0, \overline{e_2})$, $\Delta < 0$, V keeps positive for $\lambda \in (\lambda^*, 1)$;

When or $e_1 \in (\overline{e_1}, \overline{\overline{e_1}})$ and $e_2 \in (\overline{e_2}, 1)$ $\Delta > 0$, V is positive for $\lambda \in (\max[\lambda^*, \lambda_2], 1)$.

Appendix 7: Profit comparison in demand uncertainty situation

When $0 < \tilde{V} < V^*$, define the smaller real root of $A_1 c_d + A_2 c_d^2 + \tilde{V} = 0$ as $c_d^{1\sigma}$

If $c_d < c_d^{1\sigma}$, $\tilde{\Delta}\pi < 0$

If $c_d^{1\sigma} < c_d < \dfrac{a_2 dn + a_1(2b + d - dn)}{2bb_1 + d(b_1 - (b_1 + d)n)}$, $\tilde{\Delta}\pi < 0$

When $\tilde{V} > V^*$, $c_d < \frac{a_2 dn + a_1(2b+d-dn)}{2bb_1 + d(b_1 - (b_1 + d)n)}$, $\tilde{\Delta}\pi > 0$

Here we define $\overline{c_d}^{\sigma}$ as follows:

When $0 < \tilde{V} < V^*$, $\overline{c_d}^{\sigma} = c_d^{1\sigma}$;

When $\tilde{V} > V^*$, $\overline{c_d}^{\sigma} = \dfrac{a_2 dn + a_1(2b + d - dn)}{2bb_1 + d(b_1 - (b_1 + d)n)}$.

When $\tilde{V} > 0$ and $c_d < \overline{c_d}^{\sigma}$, the manufacturer's profit increases after opening a direct channel. When $\tilde{V} < 0$, the manufacturer is worse off after opening a direct channel.